An amendatory dynamic model with slip for four-wheeled omni-directional mobile robot

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ABSTRACT

Compared with the common differential driving ones, omni-directional mobile robots (OMRs) have more agilely locomotion performance, which therefore have been applied in many fields. One kind of four-wheeled OMR used in RoboCup Middle Size League competition is presented in this paper. Its kinematics and dynamic model is introduced. And considering the independent drive of four wheels which creates inevitable slipping in motion, the amendatory dynamics model that includes slipping between the wheels and the motion surface is presented. Based on the above slipping model, the velocity feedback PID controller implemented by DSP as the kernel is also given here. Experiments results demonstrate the feasibility of the dynamic model and controller.

Keywords: Dynamic Model, Omni-directional Mobile Robot, Sliding Friction Model, Motion Control

1. INTRODUCTION

Wheeled mobile robots have good maneuverability that makes them be applied widely in production and people's daily life. Differential driving is the most common movement. But with the special mechanism of omni-directional wheels, omni-directional mobile robot (OMR) performs 3 degree-of-freedom (DOFs) motion on the two-dimensional plane, which can achieve translation and rotation simultaneously along arbitrary direction. Using this type of drive system, time is saved by eliminating the need for a robot to rotate before translating from point A to point B. Due to the more agilely performance, OMRs have been applied in many fields, such as omni-directional wheelchairs[1] and in RoboCup competition[2].

Many research groups are paying attention to OMRs[3-9]. For example, Watanabe et al. presented state variable based modeling of a three-wheeled OMR, whose wheels are assumed to be symmetrically arranged orthogonal assemblies[3]. Williams et al. expanded this model to a nonsymmetrically arranged three-wheeled omni-directional mobile robot, and gave the non-linear state variable dynamics equations[8]. Muir P et al. presented the kinematics modeling of four-wheeled mobile robots[9]. From above existing research results and our experiments, we found that the three-wheeled OMRs may have stability problem due to the triangular contact area with the ground, especially when the location of the center of gravity is high or the size of the robot is small[10,11]. Therefore, it is desirable that four-wheeled vehicles will be used when stability is of great concern, for example, in the drastically antagonistic RoboCup competition environment. Therefore our first task in this article is to give the design of one four-wheeled OMR and present its kinematic and dynamic model, which was used in RoboCup Middle-Size League (MSL) competition.

However, independent drive of four wheels creates one extra DOF, which requires an accurate control for the robot. Any disturbing or disorder, even temporarily, will cause wheel slippage, while slipping is almost inherently encountered in motion for OMRs. This unexpected behavior motivated the development of a dynamic model including slip. And some researchers realized this. For example, Dickerson and Lapin present a controller for omni-directional Mecanum-wheeled vehicles, which includes wheel slip detection and compensation[4]. Williams II, Robert L. et al. presented a dynamic model for OMR, including wheel/motion surface slip and experimentally measured friction coefficients[12]. However, there are few studies on slipping model for one over constrained system of OMRs. Therefore, the main focus of the article is to present a dynamic model for one four-wheel OMR that includes slipping between the wheels and the motion surface. As one omni-directional wheel consists of a wheel hub driven by a servo DC motor and rollers that are mounted

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on the hub rotating passively, our slipping model includes both friction functions in the wheel hub rolling direction and in the roller rolling direction, which is the function of velocity in that direction respectively. Based on the above slipping model, the velocity feedback PID controller implemented by DSP as the kernel is also given here. Experiments results demonstrate the feasibility of the dynamic model and controller.

2. KINEMATICS AND DYNAMICS MODEL OF THE FOUR-WHEELED OMR

This section presents the kinematics and dynamic modeling of a four-wheeled omni-directional robot.

First we support that the robot moves in the plain face. There are two coordinate frames used in this modeling: the body frame, fixed on the moving robot with the origin in the center of gravity of the robot, denoted as $X_R O_R Y_R$, and the world frame, which is fixed on the play ground, denoted as $X_O O_O Y_O$, as shown in Figure 1.Then at any time t, the robot's state can be confirmed by the Cartesian coordinate of the robot in $\{O\}$ as

$$\boldsymbol{X}_{\mathbf{t}} = \begin{pmatrix} x_t & y_t & \theta_t \end{pmatrix}^T \tag{1}$$

The coordinate transform matrix from the body frame $\{R\}$ to the world frame $\{O\}$ is given by:

$${}^{O}_{R}\boldsymbol{T} = \begin{pmatrix} \cos\theta_t & -\sin\theta_t & 0 \\ \sin\theta_t & \cos\theta_t & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

Figure 2 shows a bottom view of our robot. Four motors are symmetrically arranged on the robot bottom so that each of wheel axes may intersect at 60 degree. L is the radial distance to the wheels from the robot center. ω_i is the rotate speed of wheel, f_i is the traction force of each wheel, whose position direction is counterclockwise.

2.1 Kinematics modeling

As shown in Figure 2, from the geometry relationship of the robot, we can get the robot's kinematics model as:

$$\boldsymbol{V}_{\omega} = \boldsymbol{J} \cdot \boldsymbol{V}_R \tag{3}$$

Where $\boldsymbol{V}_R = \begin{bmatrix} \dot{x}_t & \dot{y}_t & \dot{\theta}_t \end{bmatrix}^T$, $\boldsymbol{V}_{\omega} = \begin{bmatrix} V_1 & V_2 & V_3 & V_4 \end{bmatrix}^T = R\begin{bmatrix} \omega_{m1} & \omega_{m2} & \omega_{m3} & \omega_{m4} \end{bmatrix}^T$, R is the radius of wheels, and

$$\boldsymbol{J} = \begin{pmatrix} -1/2 & \sqrt{3}/2 & L \\ -1/2 & -\sqrt{3}/2 & L \\ 1/2 & -\sqrt{3}/2 & L \\ 1/2 & \sqrt{3}/2 & L \end{pmatrix}, \quad \boldsymbol{J}^{-1} = \frac{1}{4} \cdot \begin{pmatrix} -2 & -2 & 2 & 2 \\ 2/\sqrt{3} & -2/\sqrt{3} & -2/\sqrt{3} & 2/\sqrt{3} \\ 1/L & 1/L & 1/L & 1/L \end{pmatrix}$$

2.2 Dynamics modeling

In the body frame and by Newton's law we can get the following relationship:

$$\boldsymbol{A}\begin{pmatrix} \ddot{x}_t \\ \ddot{y}_t \\ \ddot{\theta}_t \end{pmatrix} + m\begin{pmatrix} -\dot{y}_t \\ \dot{x}_t \\ 0 \end{pmatrix} \cdot \omega = \begin{pmatrix} F_x \\ F_y \\ M_z \end{pmatrix} \quad where \ \boldsymbol{A} = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \tag{4}$$

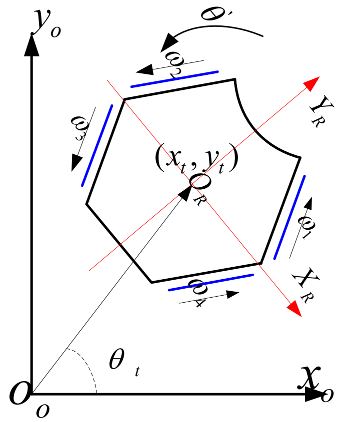
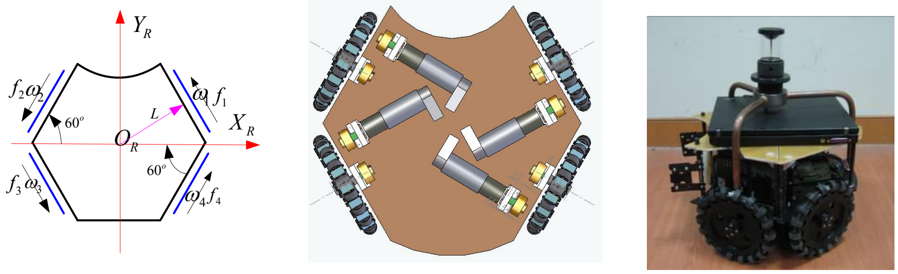


Fig. 1. The robot and the world coordinate system



(a) (b) (c)

Fig. 2. (a) Arrangement of wheels; (b) CAD model for the four-wheeled OMR; (c) Overview of the prototype robot

Here m denotes the robot's mass, I_z denotes the robot's moment inertia, F_x, F_y, M_z denotes the traction forces and the traction moment in the body frame separately, which can be derived from the geometry relationship of the robot as follows:

$$
{}_R^O\boldsymbol{T}\cdot\boldsymbol{H}\cdot\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} F_x \\ F_y \\ M_z \end{pmatrix} \quad where \ \boldsymbol{H} = \begin{pmatrix} -1/2 & -1/2 & 1/2 & 1/2 \\ \sqrt{3}/2 & -\sqrt{3}/2 & -\sqrt{3}/2 & \sqrt{3}/2 \\ L & L & L & L \end{pmatrix} \tag{5}
$$

The dynamics of each DC motor can be described as the following equations:

$$
L_a \frac{di_a}{dt} + R_a \cdot i_a + k_3 \cdot \omega_m = u \tag{6}
$$

$$
J_0 \dot{\omega}_m + b_0 \omega_m + \frac{Rf}{n} = k_2 i_a \tag{7}
$$

where L_a is the armature inductance, i_a is the armature current, R_a is the armature resistance, k_3 is the electromotive force constant, u is the applied armature voltage, J_0 is the combined moment of inertial of the motor, gear train and wheel referred to the motor shaft, b_0 is the vicious friction coefficient of the combination of the motor, gear and wheel, n is the gear ratio, f is the traction force of the wheels, k_2 is the motor torque constant， ω_m is the rotate speed of wheel.

Because the electrical time constant of the motor is much smaller than the mechanical time constant, we can neglect dynamics of the motor electric circuit, which leads to:

$$\frac{di_a}{dt} = 0\,,\, i_a = \frac{1}{R_a}(u - k_3 \cdot \omega_m) \tag{8}$$

With this assumption, and using the vector notation, the dynamics of the four identical motors can be written as:

$$J_0 \begin{pmatrix} \dot{\omega}_{m1} \\ \dot{\omega}_{m2} \\ \dot{\omega}_{m3} \\ \dot{\omega}_{m4} \end{pmatrix} + b_0 \begin{pmatrix} \omega_{m1} \\ \omega_{m2} \\ \omega_{m3} \\ \omega_{m4} \end{pmatrix} + \frac{R}{n} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} + \frac{k_2 \cdot k_3}{R_a} \begin{pmatrix} \omega_{m1} \\ \omega_{m2} \\ \omega_{m3} \\ \omega_{m4} \end{pmatrix} = \frac{k_2}{R_a} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} \tag{9}$$

Where u_i is the control input.

Combining expression (2) with (3), (4), (5) and expression (9), we can get the dynamic model of the four-wheeled mobile robot as:

$$\left(\frac{n \cdot J_0}{R^2}\, {}^{O}_{R}\boldsymbol{T} \cdot \boldsymbol{H} \cdot \boldsymbol{J} + \boldsymbol{A}\right) \begin{pmatrix} \ddot{x}_t \\ \ddot{y}_t \\ \ddot{\theta}_t \end{pmatrix} + \frac{n}{R^2}\, {}^{O}_{R}\boldsymbol{T} \cdot \boldsymbol{H} \cdot \boldsymbol{J} \cdot \left(b_0 + \frac{k_2 \cdot k_3}{R_a}\right) \begin{pmatrix} \dot{x}_t \\ \dot{y}_t \\ \dot{\theta}_t \end{pmatrix} + m \cdot \begin{pmatrix} -\dot{y}_t \\ \dot{x}_t \\ 0 \end{pmatrix} \dot{\theta}_t = \frac{k_2 \cdot n}{R_a \cdot R}\, {}^{O}_{R}\boldsymbol{T} \cdot \boldsymbol{H} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} \tag{10}$$

3. THE AMENDATORY DYNAMICS MODEL WITH SLIP

Considering the independent drive of four wheels in our robot which creates one extra DOF, one exact model and an accurate control method for the robot is required. As one omni-directional wheel consists of a wheel hub driven by a servo DC motor and rollers that are mounted on the hub rotating passively, this section presents the OMR dynamics model with slip included between the wheels and motion surface, which includes both friction functions in the wheel hub rolling direction and in the roller rolling direction, both are the function of velocity in that direction respectively. Based on the above slipping model, the robot's nonlinear dynamic model including wheel slippage avoidance constraint is presented.

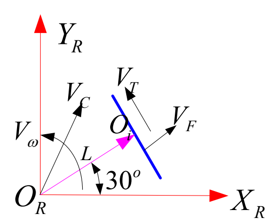


Fig. 3. Sketch map of the speed direction on one omni-directional wheel

As show in Figure 3, suppose O_i is the center of i_{th} wheel, the velocity of active roller and passive rollers in that wheel is $\vec{V}_T$ and $\vec{V}_F$ separately. O_R is the center of the robot, and its speed is $(\vec{V}_C, \vec{\omega})$, where $\vec{\omega}$ is the angular velocity , and $\vec{V}_C$ is the translational velocity. $\vec{V}_T$ is perpendicular to $\vec{V}_F$. Therefore we can gain the relationship of above velocity as follows:

$$\vec{V}_{O_i} = \vec{V}_{T_i} + \vec{V}_{F_i} = \vec{V}_c + \vec{V}_\omega \quad where \quad \vec{V}_\omega = \vec{\omega} \times \vec{L} \tag{11}$$

Let $\vec{V}_C$, $\vec{V}_\omega$, $\vec{V}_{T_i}$ and $\vec{V}_{F_i}$ project to X_R- and Y_R-axes, then we can get the relationship of above velocity as follows:

$$
\begin{cases} V_T = V_c \sin(\theta - \gamma_i)/\alpha + V_\omega \\ V_F = -V_c \cos(\theta - \gamma_i) \end{cases} \tag{12}
$$

Where θ denotes the angle between vector $\vec{V}_C$ and X_R-axis, γ_i denotes the angle between the vector $\vec{L}$ and the X_R-axis.

Therefore following the Newton's law we can get the formulas for coefficient of friction:

$$
\begin{cases} \mu_T(V_T) = \mu_{T\max} \dfrac{2}{\pi} a\tan(k \cdot V_T) \\ \mu_F(V_F) = \mu_{F\max} \dfrac{2}{\pi} a\tan(k \cdot V_F) \end{cases} \tag{13}
$$

Where $\mu_{T\max}$, $\mu_{F\max}$ is the maximum static friction coefficient in the wheel rotation and transverse direction separately, and k is the constant.

Replace b_0 in expression (10) by $\mu_T(V_T)$ and $\mu_F(V_F)$ expressed by expression (13), we can get the OMR dynamics model with slip.

4. MOTION CONTROL SYSTEM AND THE EXPERIMENTS

By using the derived dynamic model through combining expression (9) with (11) and (12), the PID based control system for the OMR has been developed, as shown in Fig. 4. This system modulates the velocity of each wheel at any moment with interpolation to achieve the given target position and velocity of the robot.

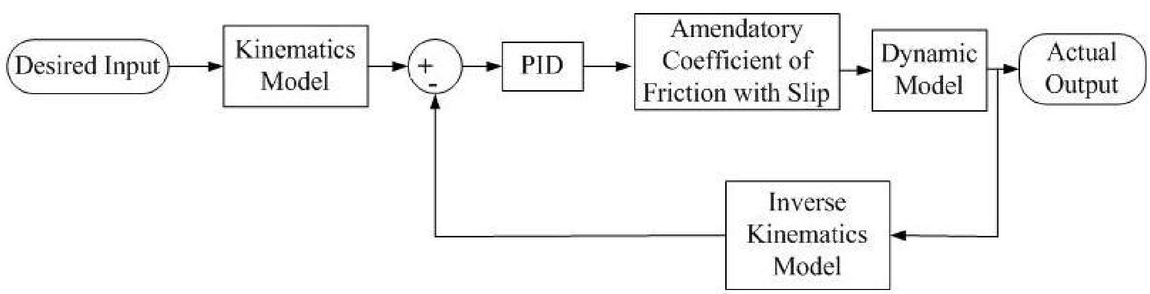


Fig. 4. Robot control structure

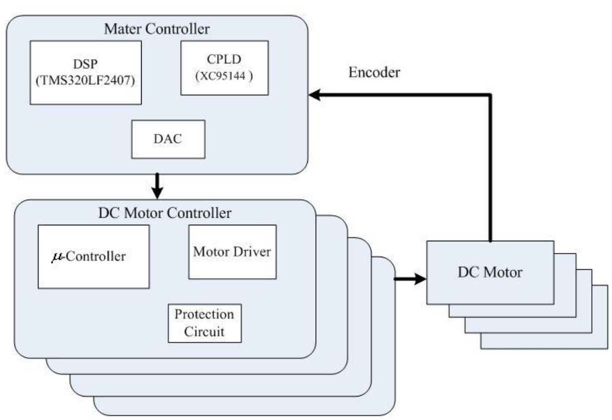


Fig. 5. Control system for the robot

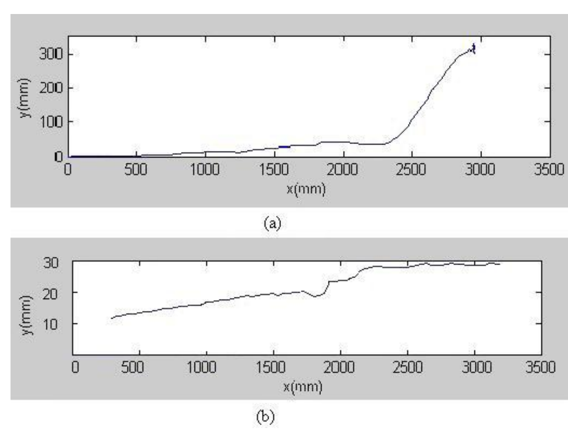


Fig. 6. Experiment data of motion

Figure 5 illustrates the control systems for the OMR. The master controller is composed of a DSP (TMS320LF 2407A) and a CPLD (XC95144), used to control the robot velocity and to command the appropriate signal depending on the feedback encoder data to the motor drivers .

In order to check the property of the robot, a pure X translational motion was commanded in experiment with the linear speed of 1500mm/s. Figure 6 shows the corresponding x and y position of the robot. Figure 6 (a) is the result without considering the effect of slipping and (b) shows the result based on the improved friction model given above. Comparing with these two experimental data, when one unexpected impact encountered in x=2350mm (Fig 6 (a)) and in x=1850mm (Fig 6 (b)) respectively, the result with the new, improved friction model showed that the slipping for Y translational motions was not as severe as the former, which demonstrates the feasibility of our analysis.

5. CONCLUSION

A holonomic omni-directional mobile robot with four independent driven omni-directional wheels was designed, and a prototype robot was developed. The kinematics and dynamic model of this robot was analyzed. Considering the independent drive of four wheels which creates inevitable slipping in motion, an amendatory dynamics model that includes slipping between the wheels and the motion surface is presented. Based on the above slipping model, the velocity feedback PID controller implemented by DSP is given here. Experiments results demonstrate the feasibility of the dynamic model and the controller.

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