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Velocity Analysis of Omnidirectional Mobile Robot and System Implementation

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***Abstract* –The kinematic modeling of an omnidirectional mobile robot is introduced, of which the arrangement of the omnidirectional wheels is arbitrary. With the particular structure of omnidirectional wheel, the kinematic performance is distinct while the robot moves in different direction, which is called anisotropy. The rule about the relationship between the maximal velocity and the amount， azimuth of the omnidirectional wheels is deduced. Especially, the kinematic performance of 4-wheeled omnidirctional mobile robot with different arrangement of the 4 wheels is analyzed. A 4-wheeled robot is designed optimally based on the result of velocity analysis. The conclusion of this paper is given and approved through experiments.**

***Index Terms* – Omnidirectional mobile robot; Kinematic modeling; Arrangement of wheels**

I. INTRODUCTION

Wheeled mobile robots have good maneuverability that makes them be applied widely in production and people's daily life. Differential driving is the most common movement. But with the special mechanism of omnidirectional wheels, omnidirectional mobile robot performs 3 degree-of-freedom (DOF) motion on the two-dimensional plane. It can achieve translation and rotation simultaneously along arbitrary direction. Any kind of motion can be implemented while keeping the pose invariable. Due to the more agilely performance, the omnidirectional mobile robot has been applied in many fields, such as omnidirectional wheelchairs[1] and RoboCup[2]. Note that the word "robot" also means "omnidirectional mobile robot" in the following paragraphs.

There were lots of discussions about omnidirectional mobile robots already. The robot with different amount of wheels has different kinetic performance, so many scholars researched into robots composed of 3-wheels, 4-wheels and 5-wheels[3][4][5]. While the position and azimuth of wheels changed, there will be different kinetic performance. Therefore the robot with a continuously variable transmission is also studied[6]. There are correlative analysis about kinematic and dynamic modeling[7][8][9]. But most of the research is about the model having fixed arrangement of wheels. In this paper, we present velocity analysis while the amount and azimuth of the wheels are both arbitrary. Also some rules about the amount and azimuth of the omnidirectional wheels are deduced.

Because the driving mechanism has important effect on the kinetic performance, analysis of the arrangement of omnidirectional wheels plays an important role in system designing. According to the characteristic of omnidirectional wheels, we deduce the kinematic equation of the robot that composed of arbitrary amount of wheels which are installed in arbitrary location of the robot body. And we also analyze what's the maximal velocity the robot can achieve when it moves in any direction. In most case, if the robot can achieve the fastest velocity is a guideline for the design. Considering the performance and the possibility of implementation, a 4-wheeled omnidirectional mobile robot is introduced, which is designed by Shanghai Jiaotong University, and the result of analysis is approved by experiment.

II. INTRODUCTION TO OMNIDIRECTIONAL WHEEL

There are many kinds of omnidirectional wheels. In this paper we discussed with Mecanum wheel which constitution is shown in Fig.1. The wheel is composed of passive rollers which are symmetrically distributed around the big active roller. It is obvious from Fig.1 that the axis(S_i) of active roller intersects the axes(E_i) of passive rollers, and the angle is α ($\alpha \neq 0$), which implies that S_i and E_i are not parallel. While the robot moves, the motor drives the active roller and the passive rollers rotate passively.

III. KINEMATIC MODELING

Robots made up of different amount (K) of wheels have different kinetic performance. With much more wheels, the robot will have much less vibration and better drive ability, but the disadvantage also exists, e.g., when $K \geq 4$, elastic mechanism is needed to keep all of the wheels contact with ground once the ground is not completely planar. So it is important to build the kinematic model for designing a robot with good performance.

Suppose there are K wheels distributed around the robot body. Fig.2 shows the correlative parameters of i th wheel. The direction of rotation speed of active roller and passive rollers is $\vec{S}_i$ and $\vec{E}_i$ respectively, and the direction of the translational speed is $\vec{T}_i$ and $\vec{F}_i$. O_i is the center of i th wheel, and its speed is $\vec{V}_{O_i}$. C is the center of the robot, and its speed is $(\vec{c}, \vec{\omega})$, where $\vec{\omega}$ is the angular velocity , and $\vec{c}$ is the translational velocity in the direction θ.

Taking no account of the performance, omnidirectional wheels can be fixed on the robot body in arbitrary position and azimuth, which means the parameters in fig.2 can be given any values. $\vec{d}_i$ denotes the vector from point C to O_i. β denotes the angle between vector $\vec{d}_i$ and

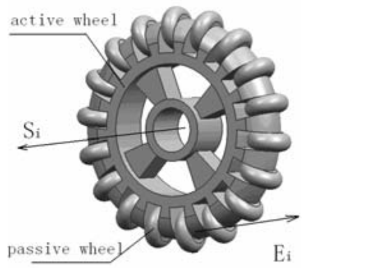

Fig.1 Sketch map of omnidirectional wheel

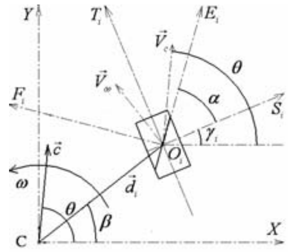

Fig.2 Parameters of i th wheel

X-axis. γ_i denotes the angle between the minus direction of active roller's angular velocity (S_i) and the X-axis. When the parameters mentioned above are fixed with some certain values, the arrangement of the omnidirectional wheels will be determined.

The velocity of wheel center O_i is determined by the velocity of active roller and passive roller (see (1)). Regarding the robot as a rigid body, we can get the velocity of O_i (shown in (2)). According to (1) and (2), (3) can be deduced.

$$\vec{V}_{O_i} = \vec{V}_{T_i} + \vec{V}_{F_i} \quad (1)$$

$$\vec{V}_{O_i} = \vec{V}_c + \vec{V}_\omega \quad \text{where} \quad \vec{V}_c = \vec{c}, \vec{V}_\omega = \vec{\omega} \times \vec{d}_i \quad (2)$$

$$\vec{V}_{T_i} + \vec{V}_{F_i} = \vec{V}_c + \vec{V}_\omega \quad (3)$$

Let V_c, V_ω, V_{T_i} and V_{F_i} project to X- and Y-axes, then we can get the relationship of the velocity along vector $\vec{T}_i$, $\vec{F}_i$ and the velocity $(\vec{c}, \vec{\omega})$ from (4) and (5). Where V_{xc} denotes the projection of $\vec{c}$ to X-axis, and with the same way, we can define $V_{x\omega}$, V_{xT}, V_{xF},

$$\begin{cases} V_{xc} + V_{x\omega} = V_c \cos\theta - V_\omega \sin\beta \\ V_{yc} + V_{y\omega} = V_c \sin\theta + V_\omega \cos\beta \\ V_{xT} + V_{xF} = -V_T \sin\gamma_i - V_F \sin(\alpha+\gamma_i) \\ V_{yT} + V_{yF} = V_T \cos\gamma_i + V_F \cos(\alpha+\gamma_i) \end{cases} \quad (4)$$

$$\begin{cases} V_T = V_c \cos(\alpha+\gamma_i-\theta)/\sin\alpha \\ \qquad + V_\omega \sin(\alpha+\gamma_i-\beta) \\ V_F = V_c \cos(\theta-\gamma_i)/\sin\alpha \\ \qquad + V_\omega \sin(\gamma_i-\beta) \end{cases} \quad (5)$$

V_{yc}, $V_{y\omega}$, V_{yT}, V_{yF}.

Due to the passive rollers not being driven by motor, V_F can be ignored during kinematic analysis. So (6) is a general kinematic equation of omnidirecional mobile robot, where V_{T_i} is the velocity of the ith wheel along $\vec{T}_i$. Obviously, once the control parameters of each wheel is given, the kinematic parameter of robot, i.e. $(\vec{c}, \vec{\omega})$, will be determined; reversibly, if $(\vec{c}, \vec{\omega})$ is known, we can deduce the angular velocity of each wheel.

$$\begin{aligned} V_{T_i} = V_c \cos(\alpha+\gamma_i-\theta)/\sin\alpha \\ + \omega d_i \sin(\alpha+\gamma_i-\beta) \end{aligned} \quad (6)$$

Where $V_{T_i} = \omega_i' r$, ω_i' is angular velocity of the ith wheel, r is radius of the wheel; V_c ($V_c = |\vec{c}|$)and ω are translational velocity and angular velocity of the robot respectively.

IV. ANALYSIS OF THE MAXIMAL VELOCITY

How fast the robot can move is a criterion for evaluating the design in some occasion. According to the special mechanism of omnidirectional wheel, analyzing the maximal magnitude of velocity is important for designing a robot with good performance. Also it is helpful for configuring the omnidirectional wheels, e.g. how many wheels will be used and where to locate the wheels, etc.

Fig.3 Omnidirectional wheel

Therefore, it is an indispensable step for studying the omnidirectional mobile robot.

During the following discussion, we use the omnidirectional wheel (see Fig.3) for analysis, which is designed in our lab. From Fig.3, it is obvious that $\alpha = 90^o$. Let the wheels distribute around the robot body symmetrically, $\beta = \gamma$, and $\vec{\omega} = 0$, which means there's only translational movement in our discussion. Then (6) can be simplified into the form of (7).

$$\begin{cases} V_{T_i} = V_c \sin(\theta - 2(i-1)\pi/K) \\ \theta \in [0, 2\pi) \\ i = [1, 2, 3 \ldots K] \end{cases} \quad (7)$$

From (7), it shows that the maximal magnitude of velocity is relative with K and θ, that means when robot moves in different direction, the maximal magnitude of velocity it can achieve is different. Due to the special mechanism of the omnidirectional wheel, the performance of the system in different direction is distinct. We call it anisotropy. To get the maximal magnitude of velocity in the direction θ of the robots which are made up of different amount of wheels, the question can be described as finding out the maximal magnitude of V_c when θ is equal to a certain value. Let M_i equals to $\sin(\theta - 2(i-1)\pi/K)$, and the maximal magnitude of $|M_i|$ is $|M_i|_{\max}$. Then , the maximal magnitude of velocity of the robot in the direction θ can be noted as $V_{c\theta}$ ($V_{c\theta} = 1/|M_i|_{\max}$, the maximal velocity of each wheel is1m/s).

For convenience, let the line joining the robot center and the first wheel center overlap with X-axis (see Fig.4).

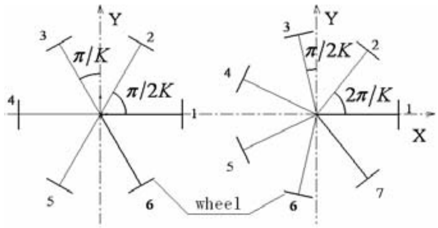

Fig.4 Arrangement diagrams of omnidirectional wheels

According to the characteristic of trigonometric function, to get the maximal magnitude of $|M_i|$ When the robot moves in the direction θ, it is equal to find out a certain value i to let (8) be tenable. In that case, the i th wheel offers the maximal angular velocity among the K wheels. According to (8), and owing to that i is integer, we can get $i = \bar{i}$ (the $\bar{i}$ th wheel rotates fastest).

$$\begin{cases} |\theta - 2(i-1)\pi/K + \pi/2| \le \pi/jK \\ |\theta - 2(i-1)\pi/K + 3\pi/2| \le \pi/jK \end{cases} \quad (8)$$

$$where \begin{cases} \text{j=1 when K is even} \\ \text{j=2 when K is odd} \end{cases}$$

Due to what mentioned above, the maximal magnitude of velocity in the direction θ, noted as $V_{c\theta}$, can be deduced (see (9)). For θ is from 0^0 to 360^0, once the (8) changes from inequation to equation, then $V_{c\theta}$ is the maximal magnitude($V_{\max}$), and θ is the direction in which the robot moves in the maximal magnitude of velocity. Then the expression of the maximal velocity $V_{\max}$ is obtained (see (10)).

$$V_{c\theta} = 1/|\sin(\theta - 2(\bar{i}-1)\pi/K)| \quad (9)$$

$$\begin{cases} V_{\max} = 1/\cos(\pi/2K) \text{ when } K \text{ is odd} \\ V_{\max} = 1/\cos(\pi/K) \text{ when } K \text{ is even} \end{cases} \quad (10)$$

From (10), $V_{\max}$ is correlative with the amount of wheels(K), and the smaller magnitude of K is , the

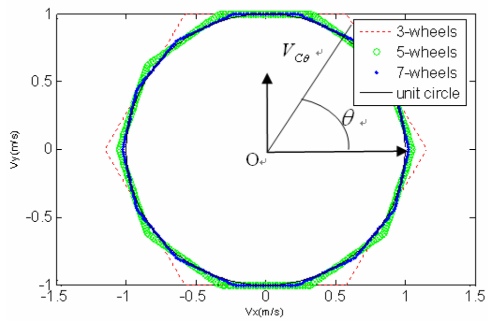

Fig.5 The maximal velocity of robot with odd wheels

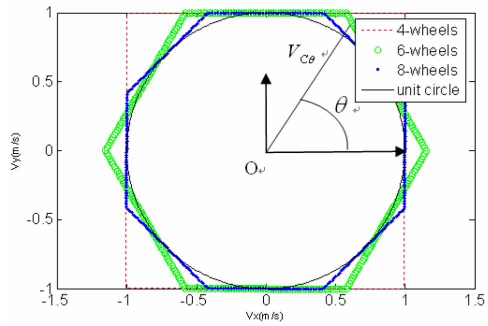

Fig.6 The maximal velocity of robot with even wheels

bigger magnitude of $V_{\max}$ will be. Thus the extremum of $V_{\max}$ will be obtained when K is 4.

To analyze the effect of wheel amount on the velocity of robot, the velocity curve is presented in Fig.5 and Fig.6. With the above analysis and information from the figure, the following rule can be deduced. The curve of maximal velocity in different direction is regular polygon, which inside meets the unit circle. As the robot constituted by even number (n) of wheels, the curve is a regular polygon with n sides. Correspondingly, with odd number (m) of wheels, it is a regular polygon with 2m sides. In the figure, point O denotes the centre of the robot, the length of radial $V_{c\theta}$ is the value of maximal velocity. Basing on this rule, what's the maximal velocity of the robot with K number of wheels moving along the direction θ is distinctly. Also we can get the conclusion that with much more wheels, the anisotropy of motion speed and drive ability is less obviously.

V. VELOCITY ANALYSIS OF 4-WHEELED OMNIDIRECTIONAL MOBILE ROBOT

In the application of omnidirectional mobile robots, 3-wheeled robots have stability problems due to the high center of gravity, and 4-wheeled robots which can achieve the extremum of $V_{\max}$ are much more stable. Therefore the 4-wheeled mechanism is in common use owing to the better performance. So we analyze the velocity of 4-wheeled robot in this section.

The kinetic performance is also distinct while the 4 wheels asymmetrically distributed around the robot body. In the following section, we will discuss the case above mentioned. Fig.7 shows 3 kinds of arrangements of 4 wheels. δ denotes the smallest angle among the 4 wheels.

According to above discussion, the kinematic equation of all the case mentioned in Fig.7 also can be expressed in (6). The value of γ_i displayed in (12). Changing (8) into the form of (13), in this inequation, we can get $i=\overline{i'}$, and the angular velocity of the $\overline{i'}$th wheel is maximal among the K wheels, i.e. $V_{c\theta}=1/\left|\sin(\theta-\gamma_{\overline{i'}})\right|$. As analyzing the case of a, b and c shown in Fig.7, the smaller δ is , the bigger maximal magnitude of velocity is. When $\delta_c = 30^o$, the maximal magnitude of velocity is 4 m/s (see Fig.8). In Fig.8, we can get a rule that the curve of the maximal velocity of 4-wheeled robots in the direction θ is a quadrangle, and the minimal internal angle equals to δ.

$$\begin{cases}\gamma_{a1}=0^o\\ \gamma_{a2}=90^o\\ \gamma_{a3}=180^o\\ \gamma_{a4}=270^o\end{cases}\quad \begin{cases}\gamma_{b1}=0^o\\ \gamma_{b2}=60^o\\ \gamma_{b3}=180^o\\ \gamma_{b4}=240^o\end{cases}\quad \begin{cases}\gamma_{c1}=0^o\\ \gamma_{c2}=30^o\\ \gamma_{c3}=180^o\\ \gamma_{c4}=210^o\end{cases} \qquad (12)$$

$$\begin{cases}\left|\theta-\gamma_i+\pi/2\right|\le A\\ \left|\theta-\gamma_i+3\pi/2\right|\le A\end{cases} \text{ where } A=(\pi-\delta)/2 \quad (13)$$

VI. IMPLEMENTATION OF 4-WHEELED ROBOT AND EXPERIMENTS

In this section, an omnidirectional mobile robot designed in our lab is introduced, and the experiments data are presented for proving the velocity analysis above.

As expressed in above section, the smaller δ is, the

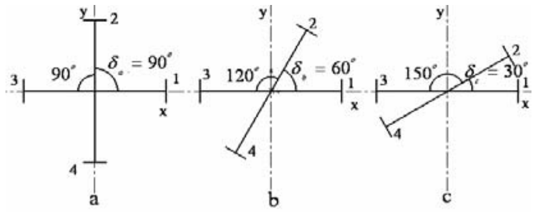

Fig.7 Arrangement diagrams of 4-wheeled robot

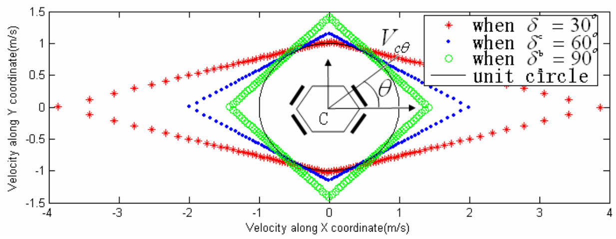

Fig.8 The maximal velocity of 4-wheeled robots in the direction θ

bigger maximal magnitude of velocity is. But with small δ, the stability of the robot will be bad, especially, when moving at fast speed and with high center of gravity. Using the omnidirectional wheel (see Fig.3) which radius is 90mm, it's hard for installation when δ is too small. Considering the analysis result about maximal velocity and practical feasibility, we design an omnidirectional mobile robot (see Fig.9). The layout of wheels can be seen in Fig.7 b, where $\delta=60^o$ .To testify the analysis above, some experiments data is present in this section.

According to (6), kinematic equation of the robot in Fig.9 can be expressed as:

$$\begin{pmatrix}V_{T1}\\ V_{T2}\\ V_{T3}\\ V_{T4}\end{pmatrix}=\begin{pmatrix}0 & 1 & R\\ -\sqrt{3}/2 & 1/2 & R\\ 0 & -1 & R\\ \sqrt{3}/2 & -1/2 & R\end{pmatrix}\cdot\begin{pmatrix}V_x\\ V_y\\ \dot{\phi}\end{pmatrix} \quad (14)$$

From Fig.8 and (14), when the angular velocity of wheels is subjected to (15), the maximal magnitude of velocity is 2m/s in the direction $\theta=30^o$; when the angular velocity of wheels is subjected to (16), the maximal magnitude of velocity is 1m/s in the direction $\theta=90^o$. In our experiments, let the velocity of wheels be subjected to (15) and (16), then find out the velocity of the robot in practice. As shown in Fig.10, the experiments data

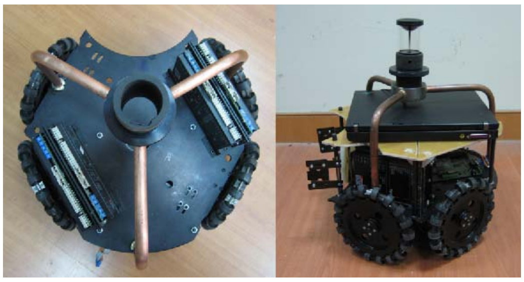

Fig.9 Omnidirectional mobile robot

accords with the velocity analysis above.

$$\begin{cases} V_{T1} = 1m/s \\ V_{T2} = -1m/s \\ V_{T3} = -1m/s \\ V_{T4} = 1m/s \end{cases} \quad (15)$$

$$\begin{cases} V_{T1} = 1m/s \\ V_{T2} = 0.5m/s \\ V_{T3} = -1m/s \\ V_{T4} = -0.5m/s \end{cases} \quad (16)$$

VII. CONCLUSION

The performance of omnidirectional mobile robot is correlative with the amount and the arrangement of the wheels, and the performance differs in different direction what we call anisotropy. The maximal magnitude of velocity that the K-wheeled robot can achieve is distinct when K changes; with the same amount of wheels, the maximal magnitude of velocity is also distinct when the robot moves in different direction or in the case that the arrangement of the wheels is different.

Because of the shortcoming of the omnidirectional wheel, the vibration will be less and driving ability will be better when the robot is composed of more wheels. According to different demand, we should determine the best scheme, e.g., how many wheels should be used to reach the best kinetic performance. To control the robot and get an optimal plan we should determine which direction the robot moves in according to the above analysis. So velocity analysis is very important for studying and designing omnidirectional mobile robots.

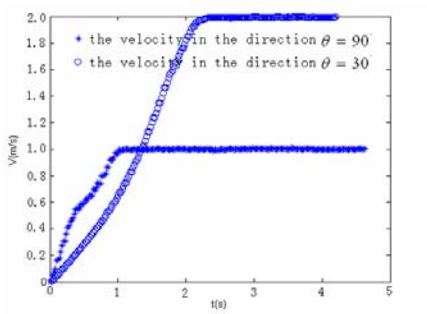

Fig.10 Experiments data

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