# Operation Principle of a Bend Enhanced Curvature Optical Fiber Sensor 

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#### Abstract

A curvature optical fiber sensor is reported in this paper. The curvature measurement sensitivity is improved using bend enhanced method. The operation principle of this intensity modulate macro-bend curvature optical fiber senor is proposed based on light scattering theory: the bend of sensitive zone brings about mode coupling and leads to the variation of surface scattering loss. The mathematic model of relationship among light loss, bending curvature, surface roughness and parameters of the fiber's configuration is also presented.


Index Terms - Curvature measurement; optical fiber sensor; operation principle; surface scattering loss.

## I. Introduction

Usually, curvature measurement is not directly made on structure, because there are few sensors available for curvature measurement. Claire Davis[1] invented a macro-bend optical fiber sensor for respirator to inspect the shape of thorax; but no treatment has been made on this sensor's surface, except fiber's macro-bend loss, so the sensitivity is very low, and it is difficult to differentiate the lead effects under bending from light loss of fiber's measurement part. In addition, it can not distinguish between positive or negative bending direction. In recent years, some scholars studied on long period optical fiber Bragg grating sensor, or interference method to measure structure curvature deformation indirectly[2]. But the measurement range of long period Bragg fiber sensor is too narrow, and it needs expensive spectrum analyser. In addition, there are micro-bend optical fiber sensors that can measure curvature directly or indirectly, but they need additional mechanical deformation device and are unfit for embedded in smart structures[3]. A curvature optical fiber sensor (COFS) is studied in this paper; it can measure bending curvature of structure directly. This COFS can be used in robot arm joint angle measurement, or tracking shape and motion of human body, it also can be used in data-glove, or applied in robot virtual reality techniques.

## II. Configuration of bend enhanced COFS

When bending radius of optical fiber is less than 10 mm , the output optical power will attenuate when the curvature become larger, but the variation is still not obvious even with small bending radius, so it is improper using macro-bending loss of fiber to measure bending deformation of structures. Specific method must be used to enhance the bending
sensitivity. The method studied in this research is to machine a sensitive zone on optical fiber's surface (see Fig. 1), then the variation of light intensity can be enlarged. The sensitive zone is a section by physical or medical treatment, which has a light emission surface along one side of the fiber only. The sensitive zone's surface must be textured, has serrations, corrugations or other texture with certain range of roughness. COFS with three forms of surface of sensitive zone are shown in Fig.1. It can be created by milling(see Fig. 2a)), grinding(see Fig. 2b)) heat forming, molding or etching. The depth of grain extend to the fiber core surface. Both plastic and glass COFS can be manufactured using those methods. Through experiments, it can be found that the effective sensitive zone's surface can be classified into tooth-like transversal grain surface, longitudinal surface, and random grain surface. All of these surface grains have good bend enhanced effect. Comparing the sensitivity of COFS with those surface untreated one, the sensitivity can be enhanced thousands of times.


Fig. 1 Three forms of sensitive zone's surface

a) Acquired by milling
b) Acquired by grinding

Fig. 2 Micrograph of the COFS's sensitive zone (100X)

## III. The existent explanations for the operation PRINCIPLE OF CURVATURE SENSOR

Up to now, about five scholars have explained the operation principle of this sensor from different points of view, and got different conclusions. But for the lack of theoretic demonstration and experimental evidence, these explanations are still in the form of hypothesis.

The operation principle explained by Lee Danisch[4] is that the sensitive zone of curvature optical fiber sensor may have effect on absorbing light, when fiber is bent to the side of sensitive zone, the rays reflecting on the sensitive zone will increase, so the transmission loss on sensitive zone's surface will increase at the same time. This theory is based on radiation law: the arbitrary surface's radiant intensity is varied with the cosine function of the angle between surface's normal direction and radiant energy transmission direction. But this conclusion contradicts with the experimental result: when bending radius is less than 100 mm , all traces of rays will meet the sensitive zone's surface at least once, then the power loss will no longer increase with bending radius decreased. The experimental result was that while the bending radius varied from 100 mm to 50 mm , the sensor's output still decreased.

Alexandar Djordjevich believed that the proportion of light rays that transmit through outer side of optical fiber will increase while bending the multimode fiber, then the transmission loss will increase for there are more rays intersect at the surface of sensitive zone[5]. But through experiments it can be found that when bending radius is larger than 10 mm , the proportion of light rays that intersect at the surface of sensitive zone are almost unchanged with the variation of the bending radius. Moreover, some conclusions deduced from Alexandar's theory of 'whisper gallery rays' are contradictive with experimental results.

Philip-Chandy have developed a fiber optic strain gauge by inserting grooves into a multimode plastic optical fiber to increase curvature measurement sensitivity[6]. This sensor's configuration was similar to optical fiber curvature sensor. Her explanation to this sensor's operation principle was that bending the fiber brought about triangle groove transformation. But when groove angle did not varied with bending radius, indicated by experimental results, the COFS still had high sensitivity.

In 2002, K S C Kuang gave a conclusion that this sensor's operation principle was that when fiber is bending, the sensitive zone's area will vary with bending, then it leads to the variation of light transmission loss[7]. This conclusion can not be approved by experiment: when fiber is bending, the variation of tensile or crushing strain on fiber's surface is very small, namely, the sensitive zone's area is almost unchanged after bending. Furthermore, according to this operation principle, it can be inferred that the sensitivity is proportional to fiber's diameter. But through experiment it can be found that when fiber's diameter becomes smaller, the relative variation of output will be decreased.

In 2001, Zhou Ling analyzed the relation between bending and light intensity loss using electromagnetic field finite element methods[8]. Since the simplified mathematical model paid no consideration to surface roughness of the sensitive zone that has the main effect to the sensitivity, the oscillating phenomena appeared in simulation results. In fact, for the stochastic surface grain on sensitive zone, it is unable to get the field solution by applying electromagnetic field finite element method to solve Maxwell's equations.

In addition, this sensor is completely different from etched cladding fiber sensor[9]. The etched cladding fiber sensor using mode mismatch principle to measure axial strain, and micro-bend principle to measure radial strain. From experiment, we can find that COFS is insensitive to extension and lateral displacement. The importance of research on the COFS's operation principle lies in improving its sensitivity, optimizing COFS's configuration and parameter of sensitive zone. In addition, distinctly differentiating this sensor from other sensors based on macro or micro-bending loss will promote this sensor into practical use and further investigation.

## IV. OPERATION PRINCIPLE OF COFS

From experimental results, COFS is only sensitive to bend deformation and insensitive to axial load. When the surface roughness is very $\operatorname{low}\left(R_{a}\right.$ less than $\left.0.6 \mu \mathrm{~m}\right)$, the sensitivity is lower either. So it can be presumed that the interface between the surface of sensitive zone and outer surrounding(air or resin) leads to waveguide interface distortion, due to surface scattering, a part of transmission mode coupled into radiation mode, and the attenuation changed with the variation of guided mode. When optical fiber is bending, the higher mode increased and more scattering loss come into existence. In this study, operation principle is presented based on theoretic analyses and a great number of experiments: transmission loss occurred due to surface scattering loss varied with bending, the variance of optical fiber's bending radius caused some of its mode power transferred, and the variation of transmission bring to surface light scattering loss.

The operation principle of COFS is similar to plane waveguide loss, and the COFS can be simplified as a plane waveguide with surface scattering loss. The transmission loss of planar waveguide is mainly from rough surface scattering loss, and the loss is increased with transmission length. Therefore, in this research, geometry optic method is used to avoid using mode theory and coupling theory that lead to boundary model ambiguity and calculation error. Then we draw an analogy between the planar waveguide transmission loss and COFS's surface scattering loss quantificationally. Light scattering on upper rough surface of planar waveguide is shown in Fig. 3. Plane wave is reflected from rough surface of planar waveguide, $n_{0}, n_{1}, n_{2}$ is the refractive index of upper cladding, waveguide, lower cladding, respectively. B is the light beam with unit length along Z axis and unit length along Y axis. The incident angle is $\theta_{i}$, the transmission power $P_{i}$ (in gauss's system of units) is[10]:

$$
\begin{equation*}
P_{i}=\frac{c}{8 \pi} n_{1} E_{y}^{2} \cos \theta_{i} \tag{1}
\end{equation*}
$$

Where, $E_{y}$ is the component of electric field $(\mathrm{V} / \mathrm{m}) ; c$ is velocity of light $(\mathrm{m} / \mathrm{s})$.


Fig. 3 Light scattering on upper rough surface of planar waveguide
According to Rayleigh criterion[10], the incident light beam scattered from upper surface of planar waveguide, and the power that reflected is $P_{r 21}$ :

$$
\begin{equation*}
P_{r 21}=\frac{c}{8 \pi} n_{1} E_{y}^{2} \cos \theta_{i} \exp \left[-\left(\frac{4 \pi \sigma_{12}}{\lambda_{1}} \cos \theta_{i}\right)^{2}\right] \tag{2}
\end{equation*}
$$

Where $\sigma_{12}$ is the mean square root deviation of waveguide's upper surface $(\mathrm{m}) ; \lambda_{1}$ is the wavelength in the medium of plane wave $(\mathrm{m}) ; \lambda_{1}=\lambda_{0} / n_{1} ; \lambda_{0}$ is wavelength in free space. Thus the power loss on unit length along Z axis is $P_{l}(\mathrm{~W})$ :

$$
\begin{equation*}
P_{i}=\frac{c}{8 \pi} n_{1} E_{y}^{2} \cos \theta_{i}\left\{1-\exp \left[-K^{2} \cos ^{2} \theta_{i}\right]\right\} \approx \frac{c}{8 \pi} n_{1} E_{y}^{2} K^{2} \cos ^{3} \theta_{i} \tag{3}
\end{equation*}
$$

Where, $K$ is the surface characteristic of planar waveguide; $K=\frac{4 \pi}{\lambda_{1}} \sigma_{12}$, then the power transmitted along Z axis is $P$ (unit width along Y axis) :

$$
\left\{\begin{array}{l}
P=\frac{c}{4 \pi} n_{1} E_{y}^{2} \sin \theta_{i} d_{e f f}  \tag{4}\\
d_{e f f}=d+\frac{1}{k_{0 x}^{\prime}}+\frac{1}{k_{2 x}^{\prime}}
\end{array}\right.
$$

Where, $d_{\text {eff }}$ is the effective thickness of planar waveguide(m). $k_{0 x}^{\prime}, k_{2 x}^{\prime}$ are the wave number in upper and lower cladding.
So the attenuation coefficient $\alpha(\mathrm{dB} / \mathrm{cm})$ is:

$$
\begin{align*}
\alpha & =\frac{1}{2} \frac{d P / d z}{P} \approx \frac{1}{2} \frac{\frac{c}{8 \pi} n_{1} E_{y}^{2} K^{2} \cos ^{3} \theta_{i}}{\frac{c}{4 \pi} n_{1} E_{y}^{2} \sin \theta_{i} d_{e f f}}  \tag{5}\\
& =217.2 \times \frac{K^{2} \cos ^{3} \theta_{i}}{\sin \theta_{i} d_{e f f}} \quad(\mathrm{~dB} / \mathrm{cm})
\end{align*}
$$

From formula (5) it can be seen that :

1) The surface scattering loss has relations with surface characteristic $K^{2}$, the attenuation is proportional to surface roughness of upper and lower waveguide.
2) The surface scattering loss has relations with incident angle $\theta_{i}$, which between light beam and planar waveguide's upper
surface, that is to say, has relation to optical mode. The more the incident angle $\theta_{i}$, the lower the guided mode, and the less the power loss. Attenuation is inversely proportional to incident angle $\theta_{i}$.
3) The surface scattering loss is proportional to the effective thickness $d_{e f f}$ of planar waveguide, the thicker the waveguide, the less the power loss.

Researchers of Bell lab have made relevant experiments to analyse the relation between optical mode and transmission loss[11]. By detecting scattered light along different path(or with different angle) pass through adjustable slit, the experimental result is that the transmission loss increased rapidly with mode increased.
D. Marcuse[12] had made researches on stochastic wall slight disturbance, and give the conclusion that relative power loss is proportional to the surface mean square deviation of stochastic wall; the loss increased while transmission mode increased; the surface scattering loss has relations with surface characteristic and transmission mode.

The above mathematic model deduced from planar waveguide can be directly applied on round optical fiber. Conclusion can be made that the operation principle of COFS is that the variation of optical fiber's macro-bending radius lead to variation of scattering loss. Consider only meridional rays, when optical fiber under macro-bending, and the sensitive zone locate at the side of convex $\operatorname{arc}$ (define this direction is positive bending) shown in Fig. 4, assumed that the left side of fiber is fixed and can not move, then the right side of fiber will change location while the bending radius of fiber becomes smaller, then the sensitive zone's surface will rotate from dash line to the location of solid line. It can be seen that when bending radius decreased, the incident angle $\theta_{i}$ between light beam and sensitive zone's surface will increase to $\theta_{i}^{\prime}$, namely, the transmission mode varied when bending, some higher guided modes coupled to lower modes, so the surface scattering loss will increase and output power will decrease. Similar analysis can be made when bending the fiber along negative bending direction, it is also in accord with above conclusion.


Fig. 4 Relation between curvature and incident angle (positive bending)
The relation between bending curvature of round fiber and incident angle can be analysed quantificationally. The optical fiber's propagation constant $\beta$ along Z axis varied with bending radius. Optical fiber has an analogy to the planar waveguide with upper surface roughened, and the surface characteristic $K$ is:

$$
\begin{equation*}
K=\left(4 \pi / \lambda_{1}\right)\left(\overline{x^{2}}-\bar{x}^{2}\right)^{1 / 2} \tag{6}
\end{equation*}
$$

Where, $x$ is the coordinate of planar waveguide surface, $\bar{x}$ is the mean value of surface coordinate. The geometrical relation between bending radius of fiber and incident angle of meridional ray is shown in Fig. 5. Suppose the diameter of fiber is $2 a$, before bending, the incident angle of meridional ray transmitted into straight fiber is $\theta$. Fasten the fiber on the left of point $P$ and bend the fiber around this point, then the bending radius of fiber on the right of point $P$ will decrease from $R$ to $r$, the incident angle of meridional ray will decrease from $\theta_{1}$ to $\theta_{2}$, incident light will intersect sensitive zone at from point $A$ to point $B$. Derived from the geometrical relation in Fig. 5:

$$
\begin{equation*}
\frac{\sin \theta}{R+a}=\frac{\sin \theta_{1}}{R-a} \tag{7}
\end{equation*}
$$

The relation between bending radius and incident angle is:

$$
\begin{equation*}
\theta_{1}=\arcsin \left(\frac{R-a}{R+a} \sin \theta\right) \tag{8}
\end{equation*}
$$

From formula (5), remove other factor that will affect attenuation, only consider the incident angle that lead to the variation of attenuation coefficient, for convenience, bring in


Fig. 5 Relation between curvature and incident angle of meridional rays dimensionless factor $\alpha^{\prime}$ ( normalized attenuation coefficient):

$$
\begin{equation*}
\alpha^{\prime}=2 \frac{d P / d z}{P} \frac{d_{e f f}}{K^{2}} \tag{9}
\end{equation*}
$$

For multimode fiber, suppose $2 a$ is the diameter of optical fiber, then $d_{e f f} \approx 2 a$, the normalized relative attenuation coefficient $\alpha^{\prime}$ is :

$$
\begin{equation*}
\alpha^{\prime}=2 \frac{d P^{\prime} d k}{P} \frac{d}{K^{2}}=\frac{\cos ^{3} \theta_{1}}{\sin \theta_{1}}=\left[1-\left(\frac{R-a}{R+a} \sin \theta\right)^{2}\right]^{3^{3 / 2}}\left(\frac{R-a}{R+a} \sin \theta\right)^{-1} \tag{10}
\end{equation*}
$$

In a similar way, when bending the fiber along negative direction, the normalized relative attenuation coefficient $\alpha^{\prime \prime}$ can be deduced:

$$
\begin{equation*}
\alpha=2 \frac{d P / d z}{P} \frac{d}{K^{2}}=\frac{\cos ^{3} \theta_{1}}{\sin \theta_{1}}=\left[1-\left(\frac{R+a}{R-a} \sin \theta\right)^{2}\right]^{3 / 2}\left(\frac{R+a}{R-a} \sin \theta\right)^{-1} \tag{11}
\end{equation*}
$$

Assume that $a=0.25 \mathrm{~mm}, \quad \theta=1.256 \mathrm{rad}$, then the simulation result of relation between normalized relative attenuation coefficient and bending radius is shown in Fig. 6 and Fig. 7. It can be seen that for the fiber only has sensitive zone on one side, when the fiber is under positive bending, the output of light power will increase with bending radius expand; when the fiber is under negative bending, the output of light power will decrease with bending radius expand.

For a light wave (propagation constant is $\beta$ ) with specific guided mode, from formula (5), (10), (11), and bring in positive and negative bending output power loss ratio $\eta_{1}$ and $\eta_{2}$, then the relation can be found between bending radius of optical fiber, optical fiber's parameters, fiber's output power $P^{\prime}$ under positive bending and output power $P^{\prime \prime}$ under negative bending. So the mathematic model of bending result in optical fiber's surface scattering loss is:


Fig. 6 Relation between positive bending and normalized attenuation factor


Fig. 7 Relation between negative bending and normalized attenuation factor

$$
\begin{align*}
& P^{\prime}=\eta_{1} P_{0} \exp \left[-\frac{1}{4} \frac{z K^{2}}{a}\left[1-\left(\frac{R-a}{R+a} \sin \theta\right)^{2}\right]^{3 / 2}\left(\frac{R-a}{R+a} \sin \theta\right)^{-1}\right]  \tag{12}\\
& P^{\prime \prime}=\eta_{2} P_{0} \exp \left[-\frac{1}{4} \frac{z K^{2}}{a}\left[1-\left(\frac{R+a}{R-a} \sin \theta\right)^{2}\right]^{3 / 2}\left(\frac{R+a}{R-a} \sin \theta\right)^{-1}\right] \tag{13}
\end{align*}
$$

Where, $P_{0}$ is the initial power of guided mode (W); $z$ is transmission distance ( mm ) ; $\theta$ is incident angle under a specific propagation constant $\beta$ (rad); $a$ is fiber's radius (mm); $\eta_{1}$ is the constant of light output loss power ratio under positive bending; $\eta_{2}$ is the constant of light output loss power ratio under negative bending.

The surface characteristic $K$ can be obtained from formula (6). Because the configuration of round optical fiber is different from planar waveguide, so the mathematic model based on planar waveguide only analyses the meridional rays, and takes no consideration on skew rays, so the mathematic model of the relationship between bending radius and fiber's parameters must be revised base on experiments and using multiple regression analysis method. Formula (12) and (13) indicated that optical power loss under bending is in relation to surface scattering loss, bending of sensitive zone lead the
variation of transmission mode and result in the variation of surface scattering loss. In addition, surface scattering loss bears a relation to sensitive zone's surface roughness and optical fiber's diameter.

## V. EXPERIMENT VERIFICATION

A considerable number of experiments had been made to analyse and verify the operation principle of COFS. In experiments, multimode plastic optical fibers and grinding method was used to acquire twelve kinds of surface roughness of sensitive zone, and then the surface roughness was measured on Taylorsurf-6 profile meter. Actual measurement result of relation between surface roughness and relative loss is shown in Fig. 8. The relative output power loss increased with surface roughness. It can be found that when the surface is relatively smooth (the critical value of $R_{a}$ is less than $0.6 \mu \mathrm{~m}$ ), there is not much variation of optical fiber's surface scattering loss under bending. After roughening treatment, the surface scattering loss will increase with surface roughness (when $R_{a}$ is greater than $6 \mu \mathrm{~m}$, the scattering loss become stable).


Fig. 8 Relation between surface roughness and relative loss
Formula (5) indicates that, the attenuation coefficient $2 \alpha$ is inversely proportional to fiber's diameter, it decreased when fiber's diameter become larger. In experiments, three fibers with different diameter were selected, maintained other parameter of optical fiber unchanged, and then measured the attenuation coefficient. The relation between fiber's diameter and relative loss is shown in table 1. It can be seen that they are inversely proportional to each other. Theoretical calculation meets the actual measurement result to a large extent. Comparison of theoretical calculation (by formula (12) and (13)) with experimental result on relative output versus bending radius of COFS is shown in Fig. 9. Theoretical calculation meets the actual measurement result to a large extent and indicates that the quantitative mathematical model is available. When bending radius is larger than 20 times of optical fiber's diameter, the macro-bending loss can be ignored and the loss comes only from scattering loss. In addition, comparison of theoretical calculation with experiment result for the COFS with sensitive zone on all circumference is shown in Fig. 10. In that case, optical power

TABLE I
ReLATION BETWEEN FIBER DIAMETER AND RELATIVE LOSS

| Fiber diameter (mm) | Output voltage without sensitive zone (V) | Output voltage with sensitive zone (V) | Power attenuation coefficient <br> ( $\mathrm{dB} / \mathrm{m}$ ) |
| :---: | :---: | :---: | :---: |
| 1.00 | 9.262 | 3.356 | 440.9 |
| 0.50 | 9.304 | 1.070 | 939.3 |
| 0.25 | 8.561 | 0.504 | 1230 |



Fig. 9 Comparison of theoretical calculation with experiment result for the output of COFS


Fig. 10 Comparison of theoretical calculation with experiment result for the COFS with sensitive zone on all circumference
output decreases under both positive and negative bending, and the simulation outcome meets with the experimental result. This phenomenon is unexplainable by Alexandar's theory of 'whisper gallery rays'.

## VI. CONCLUSION

Based on experiments and theoretical analysis, the operation principle of COFS is proposed: The light transmission loss is caused by the surface light scattering, the guided mode of the fiber varied with bending radius at the same time, and leads to the variation of surface light scattering loss. Relation between bending and power loss can be illustrated by the planar waveguide surface light scattering
theories. A mathematical model is presented on the relation of relative optical output power with bending radius and fiber's parameters. Further experiment is needed to revise and perfect this model.

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