J Intell Robot Syst (2008) 52:101–120
DOI 10.1007/s10846-008-9205-x

Modeling and Analysis of the Dynamics of an Omni-directional Mobile Manipulators System

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Received: 16 January 2007 / Accepted: 14 January 2008 /
Published online: 11 March 2008

Abstract This work studies the dynamic modeling method for a service robot with Omni-directional Mobile ManipulatorS configuration. Based on screw theory, Lie group notations, reciprocal product of twist and wrench, and Jourdain principle, the robot's motion equations including the whole body manipulation are formulated with left invariant representation. A legible and canonical dynamic model representing the relation between the inputs and the generalized dynamic load wrenches is presented. Considering the tradeoff between the symbolic concision, the modularization in code realization and the computation load, the dynamic model is decomposed into succinct block factorizations, and the basic computation unites are boiled down to the adjoint map corresponding to each joint. The traditional Lie bracket operation is extended to a generalized form. Computation efficiency, for the coefficient matrixes of the system motion equation, is discussed based on its special representation form. The generalization of the modeling method with Lie group and algebra tool is also summarized.

Keywords Mobile manipulators · Service robot · Whole body manipulation · Screw theory · Lie group · Jourdain principle · Kinematics · Dynamics

1 Introduction

Service robotics has been one of major driver for the new robot generation and the progress in sensor-based adaptivity for robotics [1]. Robot manipulators mounted on mobile platform are the familiar design for service robots, such as Robonaut [2, 3], MR Helper [4], HERMES [5], ARMAR [6] etc. In view of the dexterity, mobility and foreseeing adaptivity

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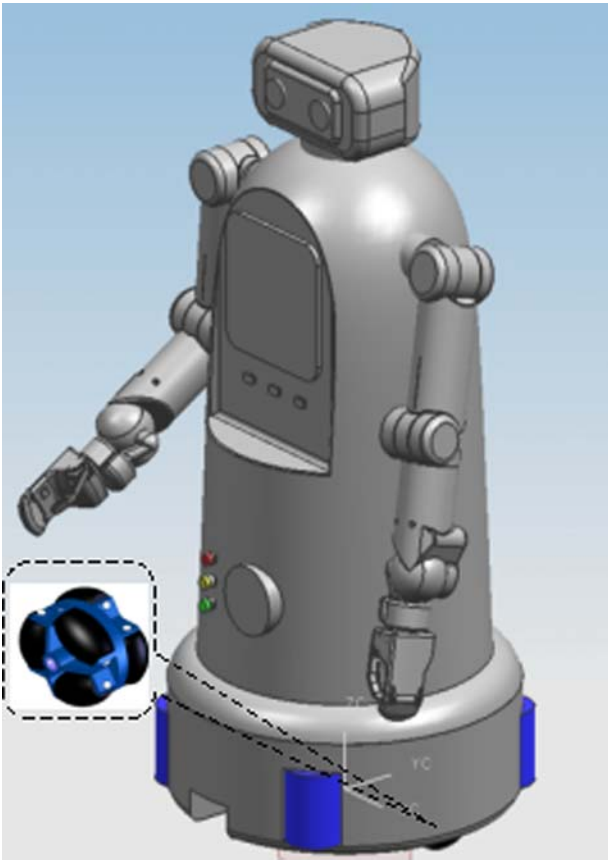
of the mobile manipulators systems, many researchers have been attracted into this subject [1–27]. Most part of known literature devoted to the mobile manipulator systems with one manipulator mounted on mobile platform, and the mobile platform is subjected to nonholonomic kinematic constraints and driven by conventional wheels [7–10, 12–21].

The mobile manipulators systems often need to fulfill the missions with contact to environment and are prone to tip over [12, 14, 18, 19], so the dynamics of whole systems must be considered in path planning [1, 12, 14], motion optimization [13, 17] and feedback control algorithm design [8–11]. The dynamic interaction simulation with contact and friction needs a systematic, compact and close form motion equation of differential system states. For achieving higher performance of robot, the model based control algorithm design is the usual way taken by researcher community [2, 22, 23]. Therefore, the compact, schematic and numerical efficiency dynamic model of complex robot systems is critical important for understanding the dynamic interaction between components of the systems, optimization design, interaction compensation and applying advanced model-based control algorithm. In this work, we will present a new simplified screw based method to model the complex multi-body system dynamics in detail, and show its application in modeling the dynamics of an Omni-directional Mobile ManipulatorS (OMMS) with two redundant manipulators mounted on the mobile platform, which is driven by three Mekanum or omni wheels. Figure 1 shows the service robot prototype.

Fig. 1 The dual arm omni-directional service robot



Let us review some related studies in literature for the mobile manipulator(s) systems. Kazuhiro et al. [4] applied the impedance method to control MR Helper, which has two 7-*dof* manipulators mounted on an omni-directional base. The coordination control for arms, base and human in object moving mission was implemented indirectly with kinematics control by adjusting or choosing the impedance parameters in serial connect bodies. The dynamic coupling or interaction between the arms and the mobile base is ignored. However, it is simple and easy to implement. Yu and Chen [7] studied the dynamic model of a nonholonomic mobile manipulator that consists of one multi-*dof* serial manipulator and a mobile platform driven by conventional wheels. They deduced the recursive velocity and acceleration equations of each component of the system in the global reference frame, and formulated the dynamic equation in variational form based on d'Alembert principal. Yamamoto and Yun [9, 10, 15] applied Lagrange method with multiplier to formulate the dynamics and studied the effect of the dynamic interaction between the manipulator and the mobile platform in the task that the end point of the manipulator tracks a desired trajectory in a fixed reference frame. They obtained the observation that the control scheme with full dynamic compensation out-performs the one without dynamic compensation by computer simulations. Cheng and Tsai [11] applied Lagrange equation with multiplier to formulate the dynamic equation of a nonholonomic wheeled mobile manipulators with two arms. The state space expression of system dynamics is obtained by eliminating the Lagrange multiplier with the null space matrix that column vectors independently span the null space of wheels' motion constraints matrix. Because the analytical solution of the null space matrix is usually not possible [25], this method cannot get the symbolic form of the dynamic equation of systems in general. Naderi, Meghdari and Durali [21] applied iterative Newton-Euler algorithm to formulate the dynamic equation of a 2-*dof* manipulator attached to a wheeled vehicle, which has a 2-*dof*-suspension system between the vehicle body and the wheels. Chen and Zalzala [26] presented an approach for the modeling and motion planning of a mobile manipulator system under nonholonomic constraint. They used the Newton-Euler equations to obtain the complete dynamics of the system. Other topics about the mobile manipulators, such as dynamic stability [12, 14, 18, 19], path planning [1, 12, 14] and kinematics [8, 16] or dynamics [25, 27] control under nonholonomic constraints, are mostly for the configuration with one manipulator mounted on conventional wheeled mobile base. Ploen and Park [28] formulated the dynamics of fix-based cooperating manipulators manipulating a common work piece using the local coordinates and the notations from the theory of Lie group. The resulting closed-form equations provide a high-level description of the motion equations that reduce the symbolic complexity. However, the whole arm manipulation and the mobile base are not discussed.

The early works for mobile manipulator(s) are mainly concerned about single manipulator mounted on a mobile platform driven by conventional wheels, and little about the one with two or more manipulators. In fact, most of service robots are usually designed to work in environment with interface for human being and are expected to be an assistant or to take place of human to do some works used to be fulfilled by human, so the structure for dual manipulators with more than 6-*dof* in one manipulator is the best and appropriate configuration design. Whole arm manipulation [29] and cooperated manipulation of manipulators are unavoidable in model the system dynamics for the dual arm service robot. We present a formulation of the dynamic equation of the dual arm mobile service robot with geometry view and better schematic based on the tool of Lie group and screw. The dynamic interaction and coupling between the arms and the mobile base have a legible form and clear physical meaning. The whole body manipulation was modeled into system dynamics with assumption that the contact forces can be detected with tactile sensors on the body and the links of arms. In view of the contradiction between the fixed reference frame and the wide space mobility of mobile service robot, we adopt the left invariant body screw coordinates (*In fact, the screw motion is expressed in an instantaneous image inertia frame that superposes with the body fixed frame. In this work, we obey the notation habits and call this image frame as body fixed coordinate frame*) to model the motion equation of the system [30]. By fully applying the operator for the Lie group $SE(3)$ and Lie algebra $se(3)$, factorizing the coefficient matrix, we reduced the symbolic complexity in the fullest extent. Numerical efficiency is obtained with proper calculation of the components in the matrix of the motion equation, the efficient product-of-exponential (POE) computation [31] and the decomposition of the coefficient matrix of motion equation. Although the driving mechanism and the dual manipulators configuration for the OMMS are special, the dynamic modeling method can be taken as a general approach for dynamics related research of the tree topology structure systems.

Geometry background for the rigid body motion and force wrench is not expatiated repeatedly for the length consideration. The reader is referred to [30, 31] and [33–35] for the detailed background information.

In "Section 2", the kinematics of the mobile platform and the manipulators are studied respectively. The velocity and acceleration are all expressed with left invariant style. "Section 3" derives the canonical dynamic model of the mobile manipulators system. The generalized form of Lie bracket operation is presented. Explanations are given to the terms that account for the dynamic interactions. Further discussions about the extensions of the modeling method are summarized in remarks. "Section 4" discusses the way to improve the numerical efficiency and summarizes the inverse dynamics computation steps. Finally, "Section 5" gives the conclusions.

2 Kinematics of the OMMS System

2.1 System Description

Figures 1 and 2 show the configuration of the OMMS system considered in this work. The system consists of three subsystems: one omni-directional mobile platform and two individual spatial 7-*dof* manipulators. The mobile platform is depicted as a trunk rigidly mounted on a triangular plate in the Fig. 2, and connects with revolute joints to three independent driving Mekanum wheels that are symmetrically configured and consist of a number of free spinning castor wheels. The castor wheels are positioned on the periphery of the Mekanum wheel's circumference and allowed for near-friction-free movement parallel to the Mehanum wheel's rotation axis. The label in Fig. 1 shows the prototype of the Mekanum wheel in this work. Two spatial multi-rigid-body manipulators are fixed rigidly on the left and right side of the shoulder respectively. The kinematics modeling method can be easily extended to multi manipulators attached on a mobile platform, though we model the kinematics of the two manipulators only.

The following notations will be used in the derivation of the kinematics and the dynamics of the OMMS system:

$W(O\text{-}X_oY_oZ_o)$	The global reference frame;
$C(O_c\text{-}X_cY_cZ_c)$	The body fixed reference frame for the trunk, the origin locates at the intersection point of the three wheel's rotation axes;
$L(O_L\text{-}X_LY_LZ_L)$	The initial tool reference frame for the left arm, it is fixed relative to the frame C;

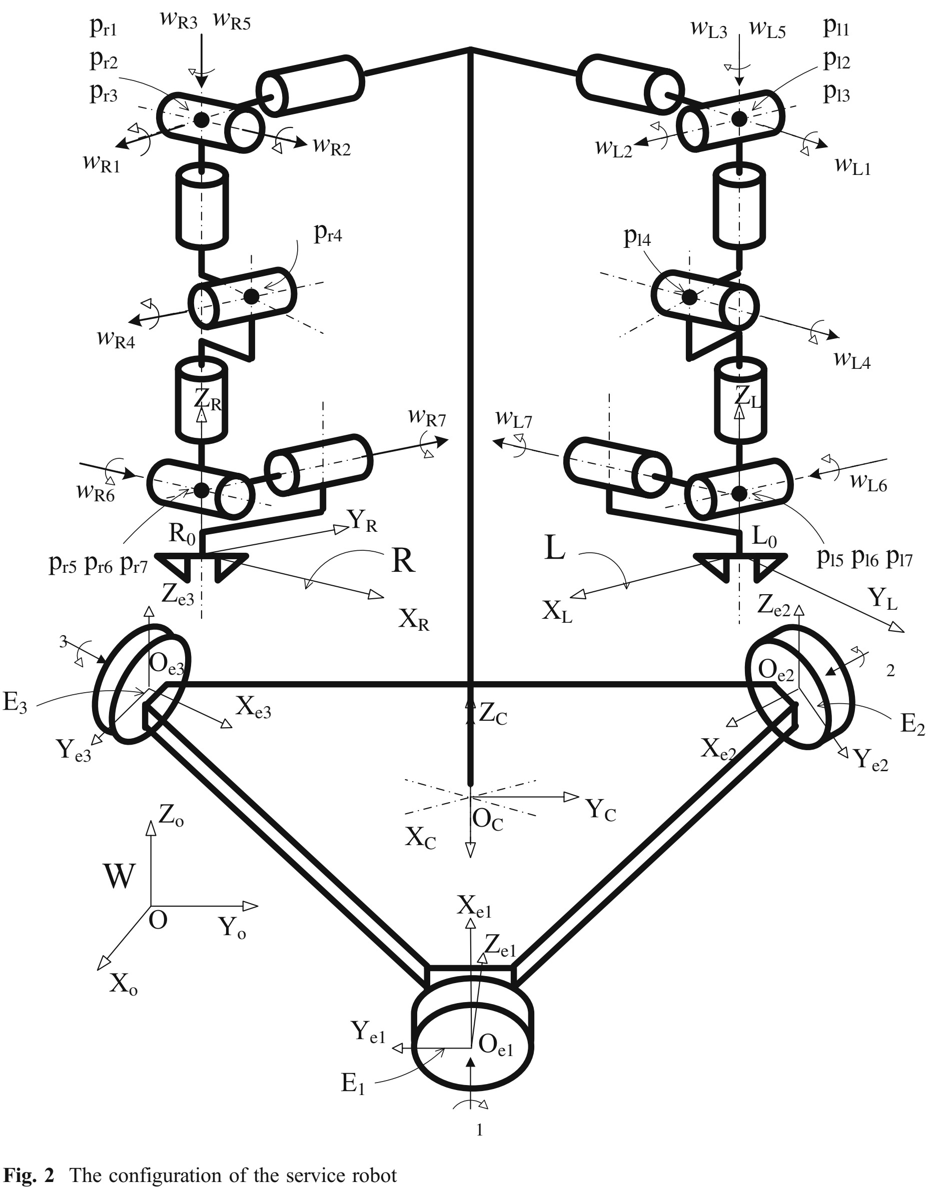


Fig. 2 The configuration of the service robot

$R(O_R\text{-}X_RY_RZ_R)$	The initial tool reference frame for the right arm, it is fixed relative to the frame C;
$L_i(O_{L_i}-X_{l_i}Y_{l_i}Z_{l_i})$	The body fixed reference frame (omitted in Fig. 2 for clarity) for the link i of the left arm, i=1,...,n;
$R_i(O_{R_i}-X_{r_i}Y_{r_i}Z_{r_i})$	The body fixed reference frame (omitted in Fig. 2 for clarity) for the link i of the right arm, i=1,...,n;
$E_i(O_{ei}-X_{e_i}Y_{e_i}Z_{e_i})$	The body fixed reference frame of the Mekanum wheels, Z_{e_i} is aligned to the rotation axis, Y_{e_i} is initially vertical to the floor, i=1,2,3;
g_{cl}, $g_{cr} \in SE(3)$	The configuration of the frame L and R relative to the frame C;

$w_{l_i}, w_{r_i} \in R^3$	The unit vectors in the direction of the joint i's axis for the left and right arm, defined in L and R respectively, i=1,...,n;
$q_{l_i}, q_{r_i} \in R$	The rotation angle of the joint i for the left and right arm, i=1,...,n;
$p_{l_i}, p_{r_i} \in R^3$	An arbitrary point on the axis of the joint i for the left arm and right arm, defined in L and R respectively, i=1,...,n;
$\xi_{l_i}, \xi_{r_i} \in se(3)$	The constant twist of the joint i of the left and right arm with screw motion at $q_{l_i} = q_{r_i} = 0$, defined in L and R respectively, i=1,...,n;
θ_i	The rotation angle of the wheel i, i=1,2,3;
a	The radius of the Mekanum wheel;
d	The distance between the intersection point of the three wheels' rotation axis and the center of the wheel;
β_i	The angle between the axis X_{e_i} and the axis X_c;
l_i=[cos β_I, sin β_i, 0]^T	The unit vector representing the direction of the origin of the E_i in the frame C, i=1,2,3 (depicted in Fig. 3);

Under the planar motion assumption, the omni-directional mobile platform has only three *dof*'s. One can choose the angles of the driving wheels as the generalized coordinates of the mobile platform for the convenience of control strategy design. The objective of the kinematic analysis of the mobile platform is to derive the kinematic equations of the mobile platform in terms of these generalized coordinates.

2.2 Kinematics of the Mobile Trunk

The omni-directional mobile platform is depicted with its planform in Fig. 3. The pose and configuration of the trunk relative to frame W has the form:

$$R_c = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \in SO(2), \quad g = \begin{bmatrix} R_c & r_c \\ 0 & 1 \end{bmatrix} \in SE(2) \tag{1}$$

where α is the angle between X_o and X_c, $r_c \in R^2$ is the position of point O_c in the plane O-X_oY_o.

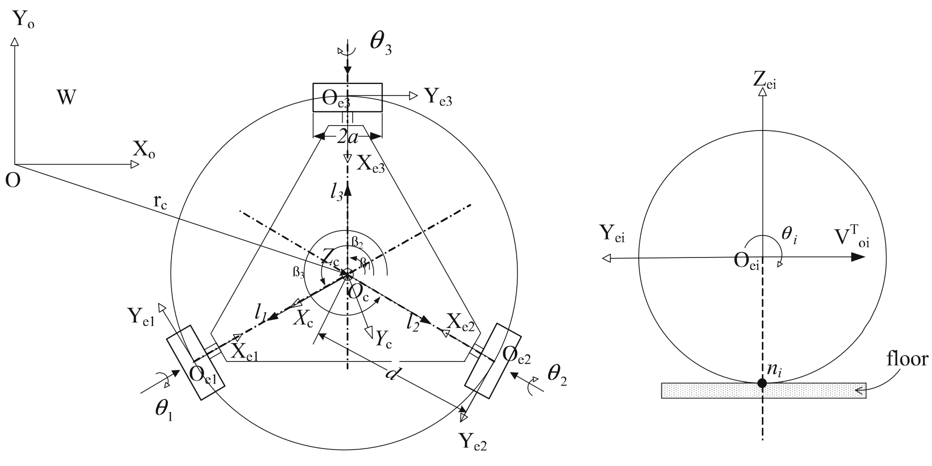


Fig. 3 The configuration of the mobile trunk and the lateral view of the wheel

The twist of the trunk with respect to the body fixed frame C can be expressed as:

$$\widehat{V}_c = g^{-1}\dot{g} = \begin{bmatrix} \widehat{w}_c & v_c \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} R_c^T\dot{R}_c & R_c^T\dot{r}_c \\ 0 & 0 \end{bmatrix} \in se(2) \tag{2}$$

Where "∧" is an operator to forms a matrix out of vector forms. All the twists are expressed in the corresponding body fixed frame with no especial specification in this work. The vector form of the trunk's twist can be represented as:

$$V_c = \begin{bmatrix} v_c \\ w_c \end{bmatrix} = \begin{bmatrix} R_c^T\dot{r}_c \\ \left(R_c^T\dot{R}_c\right)^{\vee} \end{bmatrix} \in R^3 \tag{3}$$

Where "∨" is an operator to forms a vector out of matrix forms. In following sections, the spatial form of the $V_c \in R^6$ will be often used with no special note. Reader can differentiate it with the context of its applications.

Let $q^c \in R^2$ represent the projection vector of any point within the trunk on the plane O_c-X_{cc}. The velocity of the point q^c respect to the global reference frame can be written as:

$$v_p = g\widehat{V}_c q^c \tag{4}$$

For the OMMS prototype, the rotation axis of caster wheel is perpendicular to the wheel rotation axis, so the rotation of the caster wheel will not contribute to the tangential velocity of the wheel respect to center O_c. The tangential velocity v_{oi}^T of the point O_{ei} can be easily calculated under no slipping assumption as:

$$v_{oi}^T = -a\cdot\dot{\theta}_i \tag{5}$$

Combining Eqs. 4 and 5 to get the velocity of the point O_{ei} as:

$$-a\cdot\dot{\theta}_i = \left(gRot(Z,90)\frac{\vec{r}_{oi}^{\,c}}{\|r_{oi}^c\|}\right)^T\left(g\widehat{V}_c r_{oi}^c\right) = \frac{1}{d}\begin{bmatrix} -r_{oi}^c\big|_y & r_{oi}^c\big|_x & d^2 \end{bmatrix} V_c \tag{6}$$

where $r_{oi}^c = \begin{bmatrix} r_{oi}^c\big|_x & r_{oi}^c\big|_y \end{bmatrix}^T \in R^2$ is the vector of the point O_{ei} expressed in plane O_c-X_cY_c and has homogeneous form in the second bracket.

In view of the symmetry configuration of the wheels: $r_{o_1} + r_{o_2} + r_{o_3} = 0$, we can get the relation between the wheel rotation velocity and the body twist of the trunk as:

$$\dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \frac{-1}{a\cdot d}\begin{bmatrix} -r_{o1}^c\big|_y & r_{o1}^c\big|_x & d^2 \\ -r_{o2}^c\big|_y & r_{o2}^c\big|_x & d^2 \\ -r_{o3}^c\big|_y & r_{o3}^c\big|_x & d^2 \end{bmatrix} V_c = AV_c \tag{7}$$

$$V_c = P\dot{\theta} = -\frac{a}{3d}\begin{bmatrix} -2\cdot r_{o1}^c\big|_y & -2\cdot r_{o2}^c\big|_y & -2\cdot r_{o3}^c\big|_y \\ 2\cdot r_{o1}^c\big|_x & 2\cdot r_{o1}^c\big|_x & 2\cdot r_{o1}^c\big|_x \\ 1 & 1 & 1 \end{bmatrix}\dot{\theta} \tag{8}$$

where the matrix $P = A^{-1}$ is constant. Three rows of zero vectors can be added into the matrix P to get the spatial form $P \in R^{6\times 3}$ for $V_c \in R^6$.

Differentiating Eq. 8, the acceleration twist of the trunk in body-fixed coordinates has form:

$$\dot{V}_c = P\ddot{\theta} \tag{9}$$

Remarks 2.1 Above formulations are derived with body-fixed coordinates (left invariant representation of the Lie algebra of the corresponding Lie group), so the formula is invariant to the global reference frame. For the mobile robots, path planning for a static global reference frame is not convenient and impractical. The components of the matrix A and P are the robot's structure parameters and are constant. These attributes will be convenient for the path planning of the mobile robot and make the dynamics computation efficient.

2.3 Kinematics of the Wheels

As shown in Fig. 2, the configuration of the frame E_i relative to the frame C can be represented with POE as:

$$g_{ce_i} = e^{\hat{\xi}_{ce_i}\theta_i} g_{ce_i}(0) \tag{10}$$

where $\xi_{ce_i} \in R^6$ is the twist for the screw motion of the wheel i relative to the frame C, $g_{ce_i}(0) \in SE(3)$ is the initial configuration of the frame E_i relative to the frame C. The twist ξ_{ce_i} and the initial configuration $g_{ce_i}(0)$ have simple form:

$$\xi_{ce_i} = \begin{bmatrix} 0_{3\times 1} \\ l_i \end{bmatrix} \in R^6 \tag{11}$$

$$g_{ce_i}(0) = \begin{bmatrix} Rot(Z, \beta_i) & d \cdot l_i \\ 0 & 1 \end{bmatrix} \in SE(3) \tag{12}$$

Let $Ad_{g_{ce_i}^{-1}(o)} \in R^{6\times 6}$ be the adjoint transformation associated with $g_{ce_i}^{-1}(0)$. The body twist of the wheel i relative to the frame C in E_i has form:

$$V_{ce_i} = \left(g_{ce_i}^{-1}\dot{g}_{ce_i}\right)^{\vee} = Ad_{g_{ce_i}^{-1}(0)}\xi_{ce_i}\dot{\theta}_i \tag{13}$$

The twist of the wheel i in body-fixed coordinates can be expressed with the trunk twist V_c and the twist V_{ce_i} according to the formula for the transformation of body velocities [pp. 59, 30] as:

$$V_{e_i} = Ad_{g_{ce_i}^{-1}} V_c + V_{ce_i} = Ad_{g_{ce_i}^{-1}} P\dot{\theta} + Ad_{g_{ce_i}^{-1}(0)}\xi_{ce_i}\dot{\theta}_i \tag{14}$$

Differentiating Eq. 14, the body acceleration twist of wheel i has form:

$$\dot{V}_{ei} = Ad_{g_{ce_i}^{-1}(0)}\left[Ad_{e^{-\hat{\xi}_{ce_i}\theta_i}}\left(ad_{P\dot{\theta}}\xi_{ce_i}\dot{\theta}_i + P\ddot{\theta}\right) + \xi_{ce_i}\ddot{\theta}_i\right] \tag{15}$$

where $ad_{P\dot{\theta}}\xi_{ce_i}$ is the adjoint representation of the Lie bracket $\left[V_c, \xi_{ce_i}\right]$, P has the spatial form. The acceleration twist of wheel i consists of three components; they are Coriolis, convected and relative acceleration twist respectively.

2.4 Kinematics of the Manipulators

The recursive formulation of robot kinematics for open kinematic chains has been presented in [34]. In this section, we will drive the closed form of the kinematics formulation for the mobile dual arms and present further insight into the coefficient matrixes for computation efficiency.

The expression of the initial twist for joints of manipulators can be simplified by properly choosing initial configuration and reference frame. As shown in Fig. 2, we choose the relaxing state with the arms pointing vertically down to floor as the initial configuration, and the coordinates frame L and R as the initial tool reference frame for the left and right arm respectively. The constant twists $\xi_{l_i}, \xi_{r_i} \in se(3)$ have vector form:

$$\xi_{l_i} = \begin{bmatrix} -w_{l_i} \times p_{l_i} \\ w_{l_i} \end{bmatrix} \in R^6, \quad \xi_{r_i} = \begin{bmatrix} -w_{r_i} \times p_{r_i} \\ w_{r_i} \end{bmatrix} \in R^6 \tag{16}$$

We can choose arbitrarily a body-fixed coordinates frame L_i and R_i for the link i of the left and right arm to represent the motion of the link i. The initial configuration of L_i and R_i relative to L and R is denoted with $g_{ll_i(0)}$ and $g_{rr_i}(0)$ respectively. The initial configuration of the tool frame of the left and right arm relative to L and R is unit diagonal matrix I_4. The left and right arm will have similar kinematics and dynamics formula except the subscript for their same topology structure, so following derivation will only take the left arm as example. The configuration of L_i relative to frame L has form:

$$g_{ll_i} = e^{\hat{\xi}_{l_1} q_{l_1}} e^{\hat{\xi}_{l_2} q_{l_2}} \ldots e^{\hat{\xi}_{l_i} q_{l_i}} g_{ll_i}(0) \tag{17}$$

The body twists of the link i for the left arm relative to L in L_i can be easily derived as:

$$V_{ll_i} = \left(g_{ll_i}^{-1} \dot{g}_{ll_i}\right)^{\vee} = J_{ll_i} \dot{q}_l = Ad_{g_{ll_i}^{-1}(0)} \begin{bmatrix} T_{l_1 l_i} \xi_{l_1} & \cdots & T_{l_i l_i} \xi_{l_i} & 0 & \cdots & 0 \end{bmatrix} \dot{q}_l \tag{18}$$

where J_{ll_i} is the body Jacobian matrix for frame L_i relative to L, $q_l = [q_{l_1}, q_{l_2} \cdots q_{l_n}] \in R^n$ is the joint variable for left arm, and $T_{l_i l_j}(i \leq j)$ has form (it is similar but different with the A_{ij} in [pp 176, 30], A_{ij} is the simplified form of $T_{l_i l_j}$ for the joint i):

$$T_{l_i l_j} = \begin{cases} Ad_{\left(e^{\hat{\xi}_{l_i} q_{l_i}} e^{\hat{\xi}_{l_{i+1}} q_{l_{i+1}}} \ldots e^{\hat{\xi}_{l_j} q_{l_j}}\right)^{-1}} & i < j \\ I_{4\times4} & i = j \\ 0 & i > j \end{cases} \tag{19}$$

The configuration of L_i relative to the chassis frame C can be formed as:

$$g_{cl_i} = g_{cl} g_{ll_i} \tag{20}$$

From above definition, we know configurations g_{cl}, g_{cr} and L relative to R are constant. According to the reference frame invariant attribute of body twist, we can get equations:

$$V_{cl_i} = V_{ll_i} = V_{rl_i} \tag{21}$$

where V_{cl_i} and V_{rl_i} are the body twist of Link i in frame L_i relative to frame C and R respectively.

The body twist of the link i on the left arm relative to global reference frame can be derived as:

$$V_{l_i} = Ad_{g_{cl_i}^{-1}} V_c + V_{cl_i} = Ad_{g_{ll_i}^{-1}(0)} T_{l_1 l_i} V_l + V_{ll_i} \tag{22}$$

where $V_l = Ad_{g_{cl}^{-1}} V_c$ is the body twist of the trunk-fixed frame L. Differentiating Eqn. 22, we can get the acceleration twist $\dot{V}_{l_i}$ as:

$$\dot{V}_{l_i} = Ad_{g_{ll_i}^{-1}(0)} \dot{T}_{l_1 l_i} V_l + Ad_{g_{ll_i}^{-1}(0)} T_{l_1 l_i} Ad_{g_{cl}^{-1}} P \ddot{\theta} + J_{ll_i} \ddot{q}_l + \dot{J}_{ll_i} \dot{q}_l \tag{23}$$

where

$$\left(T_{l_i l_j} \xi_{l_i}\right)' = \sum_{k=1}^{n} \frac{\partial\left(T_{l_i l_j} \xi_{l_i}\right)}{\partial q_{l_k}} \dot{q}_{l_k} = \sum_{k=i+1}^{j} T_{l_k l_j} \left[T_{l_i l_{k-1}} \xi_{l_i}, \xi_{l_k}\right] \dot{q}_{l_k} = \sum_{k=i+1}^{j} T_{l_k l_j} ad_{T_{l_i l_{k-1}} \xi_{l_i}} \xi_{l_k} \dot{q}_{l_k} \tag{24}$$

and

$$\dot{J}_{ll_i} = Ad_{g_{ll_i}^{-1}(0)} \left[\sum_{k=2}^{i} T_{l_k l_i} ad_{T_{l_1 l_{k-1}} \xi_{l_1}} \xi_{l_k} \dot{q}_{l_k} \cdots \sum_{k=i-1}^{i} T_{l_k l_i} ad_{T_{l_{i-2} l_{k-1}} \xi_{l_{i-2}}} \xi_{l_k} \dot{q}_{l_k} \;\; ad_{\xi_{l_{i-1}}} \xi_{l_i} \dot{q}_{l_i} \; 0 \cdots 0 \right] \tag{25}$$

We can get the formulas for V_{rr_i}, J_{rr_i}, $T_{r_i r_j}$ and their derivatives by simply replacing the subscript in Eqs. 17, 18, 19, 20, 21, 22, 23, 24 and 25.

Remarks 2.2 Above formulation can be extended easily to the mobile multi manipulators' kinematics. The body fixed representation makes the dynamics model more direct and clear for the simple and time invariant mass and inertia representation and transformation. Global reference frame invariant attribute will be favorable for the path planning of the end-effector of the mobile manipulators.

3 Dynamics of the OMMS

Efficient recursive Newton-Euler formulation for open chain multi-body dynamics [27] is not suitable to give the closed form dynamics model of multi manipulators manipulating a common work piece. Considering the limitation for computer in differential calculus and the complexity in the differentiating process of the energy function of a *dof* abundant dynamics system, the energy function based methods, such as Lagrange method and Appel method, are usually not a good choice for the closed form dynamics modeling [36]. We adopt the reciprocal product of twist and wrench, the variation form Jourdain principle and Newton-Euler equation to formulate the self-contained motion equation of the OMMS system in kinds of manipulation task, including the whole arm manipulation and cooperating manipulation. In view of the complexity, time changing and multiplicity of the constraints came from environment and task, modeling the kinematics constraints applied by environment or task into dynamics equation of autonomous mobile robots is not a wise choice. So we adopt the following assumptions in the modeling of mobile multi-manipulator service robot system.

- The contact and friction forces applied by environment on arms and body can be detected properly; the prerequisite tactile sensors are deployed on the surface of rigid arms and body;
- There is no slipping between the wheels and the floor;

3.1 Preliminary Concepts

Murray et al. [pp 167, 30] has deduced the Newton-Euler equation for a rigid body with twist in the body coordinate frame B, which locates at the mass center and takes the rigid body's inertial principle axes as coordinate axes. The equation has form:

$$M_B \dot{V}_B - ad_{V_B}^T M_B V_B = F_B \tag{26}$$

where $M_B \in R^{6\times6}$ is the generalized inertial matrix of the rigid body in the frame B, $V_B \in R^{6\times1}$ is the twist of the rigid body respect to the frame B, $F_B \in R^{6\times1}$ is the external wrench applied on the rigid body with respect to the frame B and consists of gravitational force and external force.

Let constant matrix $g_{BA} \in R^{4\times4}$ denotes the configuration of the arbitrarily chosen body-fixed coordinate frame A relative to the frame B, V_A and F_A be the twist and external wrench with respect to the frame A. There are transformation relations:

$$F_A = Ad_{g_{BA}}^T F_B \tag{27}$$

$$V_A = Ad_{g_{BA}}^{-1} V_B \tag{28}$$

$$M_A = Ad_{g_{BA}}^T M_B Ad_{g_{BA}} \tag{29}$$

Based on the d'Alembert principle, we define the inertial wrench $F^D = M\dot{V} - ad_V^T MV$. Combining the property that the reciprocal product of twist and wrench gives the instantaneous power and Jourdain's variation principle, we can get the equation:

$$\left(F - F^D\right) \cdot \delta V = 0 \tag{30}$$

Extending above equation to multi-rigid body systems, we can get the wrench and twist variation based Jourdain principle for multi-rigid body systems:

$$\sum_{i=1}^{n} \left(F_i + R_i - F_i^D\right) \cdot \delta V_i = \sum_{i=1}^{n} R_i \cdot \delta V_i = 0 \tag{31}$$

where F_i is the generalized external wrench (include active driving force/torque, dissipation force and external wrenches acted by environment) applied on body i, R_i is the ideal constraint wrench applied on body i, F_i^D is the inertial wrench of body i, each term is expressed with respect to a chosen body-fixed frame.

For further simplification, we decompose the generalized external wrench F_i into active driving wrench F_i^A, which is acted on the joint by driving torque/force τ_i, dissipation wrench F_i^C that is acted on the joint by friction and resistance φ_i, and the external wrench F_i^E, which is acted on rigid link by environment. Then Eq. 31 has form:

$$\sum_{i=1}^{n}\left(F_i^A + F_i^E + F_i^C - F_i^D\right)\cdot\delta V_i = 0 \tag{32}$$

Let joint variable $q = [q_1, q_2, \cdots, q_n]^T \in R^{n\times 1}$ represents the generalized coordinates of the multi-rigid body system. According to the energy conservation law, the Eq. 32 can be rewritten as:

$$\sum_{i=1}^{n}\left(\tau_i\cdot\delta\dot{q}_i + \varphi_i\cdot\delta\dot{q}_i + F_i^E\cdot\delta V_i - F_i^D\cdot\delta V_i\right) = 0 \tag{33}$$

In this work, the external wrench F_i^E includes gravity, but it is not the only way to incorporate gravity into system dynamics, Luh-Walker-Paul method [32], that imparts fictitious acceleration -g to the root of tree like multi body system, can be referred to incorporate the gravity into the inertial wrench F^D also. Here, we give the modeling method for the former one.

Except the gravity, the external wrench F^E contained the wrench acted on each body of the system by environment and is assumed to be detected with tactile skin. This assumption is mainly based on the characteristic of the diversity of the operation task and the situation that the whole body or the whole manipulators of the service robot is used as a dexterous apparatus to finish manipulation and transit task [29]. In Eqs. 32 and 33, the contacting force components of the external wrench F^E are formulated in body-fixed coordinates and easy to deal with in local processor system configured for the sole rigid link. Figure 4 shows the situation that external wrenches act on one rigid body.

Let O be the global reference frame, O' be the imaginary reference frame located at the mass center of the body and with coordinate axes aligned with the axes of the frame O (O' is omitted for figure clarity), A be the arbitrary body-fixed coordinate frame, C_0 be the coordinate frame located at the mass center of the body and with the coordinates axes aligned with the principal axes of the body, C_i(i=1,...m) be the contact coordinate frame with z-axis points in the direction of the inward surface normal at the point of the contact, g_{ac_i} be the configuration of C_i respect to the frame A, $g_{o'c}$ be the configuration of O' respect to the frame C, g_{ca} be the configuration of A respect to the frame C, m be the mass of the body.

The configuration g_{ca} can be decided off-line; g_{ac_i} can be derived with sensor information of the force acting points and the configuration of the tactile sensors by local processor. Gravity acted at the mass center of the body and can be denoted as wrench

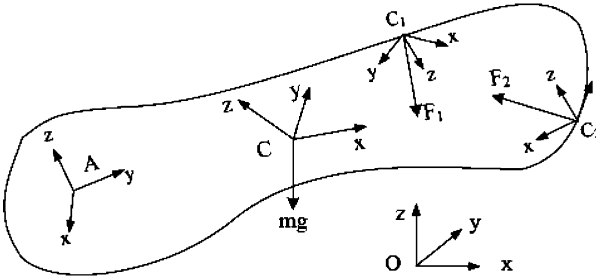


Fig. 4 External wrenches acting on one rigid body

F_g=[0,0,-mg,0,0,0] in frame O', so the external wrench acting on the body can be represented in frame A as:

$$F^E = Ad^T_{g_{o'a}} F_g + \sum_{i=1}^{m} Ad^T_{g_{c_i a}} F_i \quad (34)$$

where $g_{o'a} = \begin{bmatrix} R_{oa}R_{ac} & 0 \\ 0 & 1 \end{bmatrix} g_{ca}$, $R_{oa} \in R^{3\times3}$ is the pose matrix of the frame A respect to O and is can be extracted from the configuration of the frame A respect to O, R_{ac} can be decided off-line.

Remarks 3.1 The matrix $M \in R^{6\times6}$ can be decided off line and is constant in the dynamic model of system. So the computation burden alleviation is mainly realized by simplifying the instantaneous body twist computation process. Recursive property of adjoint transformation matrix in kinematic chain will be in favor of the computation efficiency, which will be summarized in "Section 4".

Equation 30, the system dynamics modeling principle based on second order variation form of body twist, can be used to derive all kinds of differential motion equation of multi-rigid body system subjected to first order nonlinear and nonholonomic constraints. On the other hand, screw theory based Kane style motion equation for open chain multi-rigid body system with joint variable as generalized coordinates can be easily derived as:

$$\sum_{i=1}^{n} F_i \cdot J_i - \sum_{i=1}^{n} F_i^D \cdot J_i = 0 \quad (35)$$

where J_i is the body Jacobian matrix for link i's body-fixed coordinate frame, and $\delta V = J\delta q$. The form of the system motion equation makes the modeling of the dynamics of multi-rigid body system direct and intuitive.

3.2 Dynamics Equation of the System

The OMMS system can be decomposed into three subsystems. The first is for mobile platform, which consists of three driving wheels and one rigid trunk, numbered 1–4. The second is for the left manipulator, and the last is for the right manipulator. Here we assume there are n rigid links in each manipulator. According to Eq. 33, the motion equation of the OMMS can be written as:

$$\delta q^T(\tau + \varphi) + \sum_{i=1}^{4} \left(F_i^E - F_i^D\right) \cdot \delta V_i + \sum_{i=1}^{n} \left(F_{l_i}^E - F_{l_i}^D\right) \cdot \delta V_{l_i} + \sum_{i=1}^{n} \left(F_{r_i}^E - F_{r_i}^D\right) \cdot \delta V_{r_i} = 0 \quad (36)$$

where the first term is the virtual power produced by active driving torque/force and dissipation force for the dynamics system; the second term is the virtual power produced by external wrench and inertial wrench on the mobile platform; the third term is the virtual power produced by external wrench and inertial wrench on the left manipulator; the fourth term is the virtual power produced by external wrench and inertial wrench on the right manipulator; $\tau = [\tau_1, \tau_2, \cdots \tau_{2n+3}]^T \in R^{(2n+3)\times1}$ and $\varphi = [\varphi_1, \varphi_2, \cdots \varphi_{2n+3}]^T \in R^{(2n+3)\times1}$ are the driving torque/force and dissipation force relative to active joint, $q = [\theta_1, \theta_2, \theta_3, q_{l_1}, \cdots q_{l_n}, q_{r_1}, \cdots q_{r_n}]^T \in R^{(2n+3)\times1}$ is the generalized coordinates of the mobile dual manipulators system.

Applying the mobile platform kinematics derived in "Section 2", the second term of Eq. 36 has form:

$$\delta\dot{\theta}^T\left(T_{p1}+T_{p2}\right)\left(F_p^E-F_p^D\right) \tag{37}$$

Where

$$T_{p1}=P^T\left[Ad^T_{g^{-1}_{ce_1}} \quad Ad^T_{g^{-1}_{ce_2}} \quad Ad^T_{g^{-1}_{ce_3}} \quad I_6\right]\in R^{3\times 24}$$

$$T_{p2}=\begin{bmatrix}\xi^T_{ce_1}Ad^T_{g^{-1}_{ce_1(0)}} & 0 & 0 & 0\\ 0 & \xi^T_{ce_2}Ad^T_{g^{-1}_{ce_2(0)}} & 0 & 0\\ 0 & 0 & \xi^T_{ce_3}Ad^T_{g^{-1}_{ce_3(0)}} & 0\end{bmatrix}\in R^{3\times 24}, F_p^E=\begin{bmatrix}F_1^E\\F_2^E\\F_3^E\\F_4^E\end{bmatrix}\in R^{24\times 1}, F_p^D=\begin{bmatrix}F_1^D\\F_2^D\\F_3^D\\F_4^D\end{bmatrix}\in R^{24\times 1}$$

The third term and the fourth term in Eq. 36 can be got in same way as:

$$\delta\dot{\theta}^T T_l\left(F_l^E-F_l^D\right)+\delta\dot{q}_l^T J_l\left(F_l^E-F_l^D\right) \tag{38}$$

$$\delta\dot{\theta}^T T_r\left(F_r^E-F_r^D\right)+\delta\dot{q}_r^T J_r\left(F_r^E-F_r^D\right) \tag{39}$$

Where

$$T_l=P^T Ad^T_{g^{-1}_{cl}}\left[T^T_{l_1l_1}Ad^T_{g^{-1}_{ll_1(0)}} \quad \cdots \quad T^T_{l_1l_n}Ad^T_{g^{-1}_{lln(0)}}\right]\in R^{3\times 6n}, J_l=\left[J^T_{ll_1}\cdots J^T_{ll_n}\right]\in R^{n\times 6n}$$

$$F_l^E=\begin{bmatrix}F^E_{l_1}\\ \vdots\\ F^E_{l_n}\end{bmatrix}\in R^{6n\times 1}, F_l^D=\begin{bmatrix}F^D_{l_1}\\ \vdots\\ F^D_{l_n}\end{bmatrix}\in R^{6n\times 1}, F_r^E=\begin{bmatrix}F^E_{r_1}\\ \vdots\\ F^E_{r_n}\end{bmatrix}\in R^{6n\times 1}, F_r^D=\begin{bmatrix}F^D_{r_1}\\ \vdots\\ F^D_{r_n}\end{bmatrix}\in R^{6n\times 1}$$

$$T_r=P^T Ad^T_{g^{-1}_{cr}}\left[T^T_{r_1r_1}Ad^T_{g^{-1}_{rr_1(0)}} \quad \cdots \quad T^T_{r_1r_n}Ad^T_{g^{-1}_{rrn(0)}}\right]\in R^{3\times 6n}, J_r=\left[J^T_{rr_1} \quad \cdots \quad J^T_{rr_n}\right]\in R^{n\times 6n}$$

Substituting Eqs. 37, 38 and 39 into Eq. 36, and considering the independence property of the variation of generalized coordinates, we can get the motion equation of the OMMS as a canonical form:

$$G^T\left(F^D-F^E\right)-\varphi=\tau \tag{40}$$

where

$$G=\begin{bmatrix}T^T_{p1}+T^T_{p2} & 0_{6n\times n} & 0_{6n\times n}\\ T_l^T & J_l^T & 0_{6n\times n}\\ T_r^T & 0_{6n\times n} & J_r^T\end{bmatrix}\in R^{(12n+24)\times(2n+3)}, F^E=\begin{bmatrix}F_p^E\\F_l^E\\F_r^E\end{bmatrix}\in R^{(12n+24)\times 1}$$

$$F^D=\begin{bmatrix}F_p^D\\F_l^E\\F_r^E\end{bmatrix}\in R^{(12n+24)\times 1}$$

The matrix G is the stacked form of the generalized Jacobian matrix in body coordinates for the OMMS. We have defined the d'Alembert inertial wrench of rigid body; it can be easily extended to inertial wrench vector of multi rigid body systems. Further derivation of Eq. 40 to the configuration space of the system has form:

$$\underbrace{G^T M G}_{L(q)} \ddot{q} + \underbrace{\left(G^T M H - G^T A M G\right)}_{K(q,\dot{q})} \dot{q} - G^T F^E - \varphi = \tau \tag{41}$$

where

$$M = diag[M_p \quad M_l \quad M_r] \in R^{(12n+24)\times(12n+24)} M_p = diag[M_{p_1} \quad \cdots \quad M_{p_4}] \in R^{24\times 24}$$
$$M_l = diag[M_{l_1} \quad \cdots \quad M_{l_n}] \in R^{6n\times 6n} M_r = diag[M_{r_1} \quad \cdots \quad M_{r_n}] \in R^{6n\times 6n}$$
$$H = \dot{G} = \begin{bmatrix} H_p & 0 & 0 \\ H_{l_p} & H_l & 0 \\ H_{r_p} & 0 & H_r \end{bmatrix} \in R^{(12n+24)\times(2n+3)}$$
$$H_p = \left[\left(Ad_{g_{ce_1}^{-1}} ad_P \xi_{ce_1} \dot{\theta}_1\right)^T \quad \left(Ad_{g_{ce_2}^{-1}} ad_P \xi_{ce_2} \dot{\theta}_2\right)^T \quad \left(Ad_{g_{ce_3}^{-1}} ad_P \xi_{ce_3} \dot{\theta}_3\right)^T \quad 0_{6\times 3}^T \right]^T \in R^{24\times 3}$$
$$H_{l_p} = \begin{bmatrix} Ad_{g_{ll_1(0)}^{-1}} \left(T_{l_1 l_1} Ad_{g_{cl}^{-1}} P\right)' \\ \vdots \\ Ad_{g_{ll_n(0)}^{-1}} \left(T_{l_1 l_n} Ad_{g_{cl}^{-1}} P\right)' \end{bmatrix} \in R^{6n\times 3}, H_{r_p} = \begin{bmatrix} Ad_{g_{rr_1(0)}^{-1}} \left(T_{r_1 r_1} Ad_{g_{cr}^{-1}} P\right)' \\ \vdots \\ Ad_{g_{rr_n(0)}^{-1}} \left(T_{r_1 r_n} Ad_{g_{cr}^{-1}} P\right)' \end{bmatrix} \in R^{6n\times 3}$$
$$H_l = \left[\dot{J}_{ll_1}^T \quad \dot{J}_{ll_2}^T \quad \cdots \quad \dot{J}_{ll_n}^T \right]^T \in R^{6n\times n}, H_r = \left[\dot{J}_{rr_1}^T \quad \dot{J}_{rr_2}^T \quad \cdots \quad \dot{J}_{rr_n}^T \right]^T \in R^{6n\times n}$$
$$A = diag\left[ad_{V_{p_1}}^T \quad \cdots \quad ad_{V_{p_4}}^T \quad ad_{V_{l_1}}^T \quad \cdots \quad ad_{V_{l_n}}^T \quad ad_{V_{r_1}}^T \quad \cdots \quad ad_{V_{r_n}}^T \right] \in R^{(12n+24)\times(12n+24)}$$

In matrix H_p, H_{l_p} and H_{r_p}, we extend the adjoint representation of Lie bracket to a generalized form with $ad_p\xi$. Assuming $P \in R^{6\times n}$ and has form:

$$P = \begin{bmatrix} p_{11} & \cdots & p_{n1} \\ p_{12} & \cdots & p_{n2} \end{bmatrix}, p_{ij} \in R^{3\times 1} (i = 1, ..., n; j = 1, 2)$$

Then we have:

$$ad_p\xi = \left[\begin{bmatrix} \widehat{p}_{12} & \widehat{p}_{11} \\ 0_{3\times 3} & \widehat{p}_{12} \end{bmatrix} \xi \quad \cdots \quad \begin{bmatrix} \widehat{p}_{n2} & \widehat{p}_{n1} \\ & \widehat{p}_{n2} \end{bmatrix} \xi \right] \in R^{6\times 6n}, \xi \in R^{6\times 1} \tag{42}$$

In Eq. 41, $L(q) \in R^{(2n+3)\times(2n+3)}$ is the generalized system inertia matrix, and $K(q, \dot{q}) \in R^{(2n+3)\times(2n+3)}$ is the generalized Coriolis and centrifugal force term according to the system. $G^T F^E$ includes external forces/torques, which act on the bodies of the system. As well known, the symmetric and positive definite property of the matrix $L(q)$ and the skew-symmetric property of the matrix $\dot{L}(q) - 2K(q, \dot{q})$ are the important structural properties of the motion equations of multi-rigid body dynamics system for the control law design and the convergence of the control law. Simple derivation can prove that Eq. 41 satisfies above properties.

The generalized system inertia matrix $L(q)$ contained the information about the inertia interaction of different part of the system. The matrix $L(q)$ has form:

$$L(q) = \begin{bmatrix} (T_{P_1}+T_{P_2})M_P(T_{P_1}+T_{P_2})^T + T_l M_l T_l^T + T_r M_r T_r^T & T_l M_l J_l^T & T_r M_r J_r^T \\ J_l M_l T_l^T & J_l M_l J_l^T & 0 \\ J_r M_r T_r^T & 0 & J_r M_r J_r^T \end{bmatrix}$$

$$= \begin{bmatrix} M_{Gp} & M_{G_{pl}}^T & M_{G_{pr}}^T \\ M_{G_{pl}} & M_{G_l} & 0 \\ M_{G_{pr}} & 0 & M_{G_r} \end{bmatrix} \quad (43)$$

Where the matrix $M_{G_p} \in R^{3\times3}$ is the equivalent generalized system inertia matrix on the three driving joints of the wheels, the matrix $M_{G_{pl}} \in R^{n\times3}$ and $M_{G_{pr}} \in R^{n\times3}$ are the generalized interaction inertia matrix for the mobile platform movement and the manipulators' movement, the matrix $M_{G_l} \in R^{n\times n}$ and $M_{G_r} \in R^{n\times n}$ are the generalized inertia matrix for the left and right manipulator respectively.

Similar information about the Coriolis and centrifugal force term caused by the movement of components of the system can be observed in the detailed structure of the matrix $K(q,\dot{q})$. The detailed discussion is omitted for length consideration.

Remarks 3.2 Neglecting the dissipation force in joints, the modeling process and the form of Eq. 40 revealed that we can set up the dynamics model of the active driving multi-rigid body dynamics system with a generalized approach based on screw theory. Let τ denoting the driving torque/force vector of the system, then only three components, stacked Jacobian matrix G in body coordinates, stacked external wrench F^E and the stacked d'Alembert inertia wrench F^D for each rigid body of the system need to be derived carefully to model the system dynamics, which has a canonical form:

$$G^T\left(F^D - F^E\right) = \tau \quad (44)$$

Above formula is more attractive than traditional dynamics modeling method, such as Lagrange equation, Newton-Euler equation and Kane method, in setting up the closed form dynamics model of the multi-rigid body system. Its virtue lies on the directness in reflecting the relationship of the input and the output, the succinct factorization of matrix structure, convenience in realization with computer and the concise form. For closed loop structure or parallel mechanism, only the form of the Jacobian matrix G needs to be adjusted according to different generalized coordinates selection strategy and kinematics structure.

4 Computation Aspects

The dynamic model Eq. 41 is mainly for the purpose of inverse dynamics computation of the mobile multi manipulators system and not for simulation. The reason is that the external contact wrenches cannot be resolved by only considering the self-governed robot system dynamics. It must be resolved by considering all of the entity's dynamics, which contact with the robot system in environment [37]. Assuming no or known external forces/torques act on the bodies of the system and no slippage between the wheels and floor, Eq. 41 can be used to simulate the dynamic behavior of the system under driving forces/torques.

In model-based dynamics control law design and dynamic simulation of the system, the computation efficiency of the dynamic model is the one of the important issues for the real time implementation and the modeling method selection. Park [31] had pointed out that the POE formula is not only computationally effective tool but also an ideal modeling tool for its compact form, Lie theoretic foundations, easy geometric visualization, and device-independent features respect to the DH-based kinematic representations.

As well known, the recursive Newton-Euler algorithm developed by Luh et al. [32] is very efficient in inverse dynamics computation for the serial manipulator. Though we modeled the system dynamics as closed form not recursive one with screw theory, the computation efficiency can be improved largely by properly computing the coefficient matrixes of the system dynamics equation. Observing the matrixes G, H and A in Eq. 41, we can found that the adjoint transformation matrixes are the main element for the system dynamics equation. Further decomposition of the manipulator Jacobian matrix can give deep insight of its components. Stacking Eq. 18 into matrix form as:

$$V_{ll} = \begin{bmatrix} V_{ll_1}^T & V_{ll_2}^T & \cdots & V_{ll_n}^T \end{bmatrix}^T = A(0) T_{ll} \dot{q}_l \tag{45}$$

Where $A(0) = diag\left[Ad_{g_{ll_1(0)}^{-1}}, Ad_{g_{ll_2(0)}^{-1}}, ..., Ad_{g_{ll_n(0)}^{-1}}\right] \in R^{6n\times 6n}$ is time invariant, and

$$T_{ll} = \begin{bmatrix} \xi_{l_1} & & & & \\ T_{l_1 l_2}\xi_{l_1} & \xi_{l_2} & & 0 & \\ \vdots & \vdots & \ddots & & \\ T_{l_1 l_{n-1}}\xi_{l_1} & \cdots & T_{l_{n-2} l_{n-1}}\xi_{l_{n-2}} & \xi_{l_{n-1}} & \\ T_{l_1 l_n}\xi_{l_1} & T_{l_2 l_n}\xi_{l_2} & \cdots & T_{l_{n-1} l_n}\xi_{l_{n-1}} & \xi_{l_n} \end{bmatrix} \in R^{6n\times n} \tag{46}$$

Similarly we can get the stacked form body twist vector for the left manipulator as:

$$V_l = A(0) T_{lc} + V_{ll} \tag{47}$$

where $T_{lc} = \left[\left(Ad_{g_{cl}^{-1}} V_c\right)^T \quad \left(T_{l_1 l_2} Ad_{g_{cl}^{-1}} V_c\right)^T \quad \cdots \quad \left(T_{l_1 l_n} Ad_{g_{cl}^{-1}} V_c\right)^T \right]^T \in R^{6n\times 1}$. We can found that the components $T_{l_i l_j}(i = 1, ..., n; j = 1, ..., n; i < j)$ and time invariant twist $\xi_{l_i}(i = 1, ..., n)$ in matrix T_{ll} can be used to assemble the components of H with the corresponding adjoint representation of Lie bracket $ad_{T_{l_i l_{j-1}}\xi_{l_i}}\xi_{l_j}$. So we only focus on the computation efficiency of the matrix T_{ll}. Further decomposition of the matrix $T_{l_i l_j}$ as:

$$T_{l_i l_j} = Ad_{\left(e^{\hat{\xi}_{l_i} q_{l_i}} e^{\hat{\xi}_{l_{i+1}} q_{l_{i+1}}} \cdots e^{\hat{\xi}_{l_j} q_{l_j}}\right)^{-1}} = Ad_{e^{-\hat{\xi}_{l_j} q_{l_j}}} Ad_{e^{-\hat{\xi}_{l_{j-1}} q_{l_{j-1}}}} \cdots Ad_{e^{-\hat{\xi}_{l_i} q_{l_i}}} \tag{48}$$

Equation 48 shows that the $\frac{1}{2}n(n-1)$ components of the matrix T_{ll} are the products of the time invariant twists ξ_{l_i} and the combined product $T_{l_i l_j}(i < j)$ from n elementary adjoint transformation matrixes $Ad_{e^{-\hat{\xi}_{l_i} q_{l_i}}}(i = 1, ..., n)$. For an *n-linked* serial open chain manipulator, the term $T_{l_i l_j}(i < j)$ can be reused and computed with n-1 product operations based on the n elementary transformation matrixes. Then, product operations between $T_{l_i l_j}$ and ξ_{l_i} can be implemented to get the end-effector Jacobian matrix. If we neglect the computation burden for the exponential map and the transformation from exponential map to adjoint representation, we can verify easily the algorithm for the end-effector Jocobian matrix $J_{ll_n}(J_{rr_n})$ is $O(N)$. In Eq. 41, The components $T_{l_i l_j}(i < j)$ and $T_{l_i l_j}\xi_{l_i}$ can be reused to compute the matrix H and A. Therefore the system dynamics computation can be finished efficiently with above computation mechanism.

The forward dynamics can be resolved by standard numerical integration methods with the proper initial conditions. The inverse dynamics revolution is summarized with five steps as:

Step 1 Compute and assemble the time invariant matrixes and the vectors *M*, which include P, ξ_{ce_k}, ξ_{l_i}, ξ_{r_i}, the initial configuration matrixes, such as $g_{ce_k}(0)$, g_{cl}, g_{cr}, $g_{ll_i}(0)$, $g_{rr_i}(0)$, and the corresponding adjoint transformation matrixes $Ad_{g^{-1}}$ for each initial configurations, where k=1,2,3 and i=1,...,n;

Step 2 At current configuration q, compute the exponential maps $e^{\hat{\xi}_{ce_k}\theta_k}$,$e^{\hat{\xi}_{l_i}q_{l_i}}$, $e^{\hat{\xi}_{r_i}q_{r_i}}$ and transform them to its corresponding adjoint transformation matrixes; finish the tactile sensor information processing and gravity transformation to get F^E;

Step 3 To get each body twist Vx (x=p_1,...,p_4,l_1,...,l_n,r_1,...,r_n), the matrix A, G and H with afore mentioned computation mechanism in current q and $\dot{q}$;

Step 4 Compute the dissipation forces/torques acted on joints and then apply Eq. 41 to get the driving torques/forces;

Step 5 Update $q, \dot{q}, \ddot{q}$ and then repeat step 2–4.

5 Summary

This work presents a general formulation method to model the system dynamics of an omni-directional mobile manipulators system. The method exhibits geometry view and better schematic property by using the tools of Lie group and screw. We combined the reciprocal product of the twist and wrench with Jourdain variation principle to establish a canonical dynamics model that represents directly the relation between the input and the resultant external and inertial wrench. The canonical dynamics model has concise form and is intuitive in reflecting the dynamic relationship between the input and the output.

In view of the contradiction between fixed reference frame and the wide space mobility of mobile robots, the coordinate-invariant (left invariant representation in Lie theory) representation is adopted to model the motion equation of the system. The dynamic interactions and couplings between arms and mobile base have legible form and clear physical meaning. The whole body manipulation is modeled into system dynamics with the assumption that contact forces can be detected with the tactile sensors on the surface of the trunk and links of arms.

By fully factorizing the coefficient matrixes and applying the operators from Lie group *SE*(3) and Lie algebra *se*(3), the symbolic complexity of the dynamic model is reduced in the fullest extent. The Lie bracket operation $[X, Y], X \in R^{6\times1}, Y \in R^{6\times1}$, is extended to the generalized form $[U, V], U \in R^{6\times n}, V \in R^{6\times1}$. Computation efficiency can be obtained with proper calculation of the components in the coefficient matrixes of the motion equations. The local link related modeling way and decomposition of the components can be utilized properly and conveniently in distributed control system realization. Although the driving mechanism for the prototype in this work is special, the dynamic modeling method can be taken as a general approach to model the dynamics of the tree topology structure systems with one or more branches. Some discussion and extension about the presented method are presented in remarks.

The modeling method utilizes fully the POE formula, linear algebra, and factorization of the matrixes with symbol form, so the virtues in compact form, Lie theoretic foundations, and device-independent features over the DH-based kinematics representations are shown well.

Acknowledgment The work presented in this paper is supported by the 863 National High-Tech Program of China (2006AA04Z261). At the same time, we would like to express our gratitude to YASKAWA Electric Cooperation for offering the collaborative research support.

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